INTERIM REPORT

FEASIBILITY STUDY ON THRUST-AUGMENTED, SWING-BY, INTERPLANETARY MISSIONS

by

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By

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MARSHALL SPACE FLIGHT CENTER
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FOREWARD

The research presented here was performed for the Astrionics Laboratory of the George C. Marshall Space Flight Center, Huntsville, Alabama. The contents represent work done to date on NASA Contract NAS8-20358 and fulfills the contractural obligation of a midterm progress report. This portion of Contract NAS8-20358 is devoted primarily toward research into the feasibility of using thrust agumented maneuvers on a swing-by interplanetary mission.

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SECTION 1

INTRODUCTION

This document describes the results obtained to date on the investigation of Thrust Augmented Swing-by Interplanetary Missions. The contents are not to be interpreted as fulfilling the contractual obligation on this study but rather as a mid-term progress report.

Emphasis, in this report, has been placed on the capability generated and currently being used to obtain results in the study. An example case is presented (A Launch from Mars on March 2, 1978) in terms of program output and Stromberg-Carlson plots (i.e., SC-4020 output). Tentative conclusions and discussion of the work to be performed during the remainder of the contract are also included. Future work areas are indicated.

SECTION 2

STUDY OBJECTIVES

The primary objective of the study is to access the influence of thrust augmented maneuvers on; total mission time, mission characteristic velocity. Earth-entry velocity, Mars departure velocity, and payload on orbit at Mars. Should thrust augmented maneuvers alter any of the above mentioned mission characteristics in a favorable manner and by a "sufficient" amount; the thrust augmented maneuvers would be deemed feasible for interplanetary swing-by missions. The initial basis for comparison of power assisted swing-by trajectories will be that of the gravity assisted maneuver only (i.e. pure swing-by). Another basis of comparison will be the direct return mission; that is, return to Earth directly from Mars. Only a limited amount of comparison has been accomplished on the latter basis. This task remains as one to complete in the Phase II effort of the contract.

Another objective and remaining task is to combine or place into an acceptable format the information generated on power assisted swing-bys. The chosen format must also tie together both aspects of the entire mission: the Earth to Mars phase and the Mars to Earth phase. One possible format for chart arrangement would have Mars arrival or departure data on the absissa and the magnitude of the characteristic velocity required (each phase of the mission) on the ordinate. In such a format, the stay time in park orbit at Mars would be the variable tying together the most favorable Earth to Mars and Mars-Venus-Earth "legs". More is said of this subject in the following section.

SECTION 3

STUDY CONSTRAINTS AND GUIDELINES

In order to derive usable information from the study, the swing-by leg on the return mission must be matched with the outbound leg in such a manner that the performance indices selected for weighting desirable missions reflect the appropriate penalty incurred over the entire mission. The penalty could be any number of functions of fuel expenditure, payload at Mars, weight on return to Earth, Earth entry velocity, etc. The performance index chosen for the study will be discussed later.

Useful information is also a function of the parameters or the model considered to adequately define the physical system.

The parameters considered in the study can be summarized as

1. $\Delta V_{D\oplus}$ Launch velocity (i.e., launch out of a 184 Km geocentric orbit),
2. $LD_{\oplus}$ Launch date (i.e., date of launch out of geocentric park orbit).
3. T_1 Flight time - leg 1 (time of flight from Earth park orbit departure to Mars park orbit injection).
4. $h♂$ Park orbit Altitude (park orbit altitude at Mars).
5. e Park orbit eccentricity.
6. i Park orbit orientation.
7. $\Delta V_{A♂}$ Park orbit injection velocity (injection velocity required to enter into park orbit at Mars).
8. T Stay time in park orbit at Mars.
9. $\Delta\vec{V}_{D♂}$ Departure Maneuver (maneuver out of park orbit at Mars).
10. T_2 Flight time on second leg (time from park orbit departure at Mars to Venus encounter).

11. T_3 Flight time on third leg (time from Venus encounter to park orbit injection,

or $T_3 = T_F - T_2$

T_F Total flight time on legs 1 and 2.

12. $\Delta V_{♀}$ ΔV Maneuver required at Venus.

13. $q^{♀}$ Closest Approach Distance to Venus.

14. $\Delta V_{A\oplus}$ Velocity increment required to circularize at Earth return.

15. h_g Park orbit altitude at Earth.

16. $e_{\oplus}$ Park orbit eccentricity at Earth.

The list of total parameters can be shortened by constraining the study to the following special cases (parameter constraints).

i Park orbit altitude at Mars equals a constant (i.e., $h♂ \underline{\Delta}$ 100 Km).

ii Closest approach radius at Venus greater than or equal to a constant (i.e., a grazing trajectory $q_{♀}$ = 6050 Km).

iii Circular park orbit at both departure and Earth return (i.e., $h_{\oplus}$ = 184 Km). For aerodynamic braking maneuvers, the vehicle was assumed to perform a direct entry.

The list of parameter constraints will increase as the performance indices are defined. The performance indices considered are:

1. min $[\Delta V_{♀}]$
2. min $[\Delta V_{♀} + \Delta V_{D♂}]$
3. min $[\Delta V_{\circ} + \Delta V_{D♂} + \Delta V_{A\oplus}]$.

The first performance index will be investigated to determine the nature of the minimum, if one exists over the range of variables considered. The second and third index will be investigated, taking into account the magnitudes of $\Delta V_{D♂}$ and $\Delta V_{A\oplus}$ associated with each minimum. (That is, they will be weighted according to the magnitude of the thrusting maneuver at Venus.

More of the parameters may be eliminated from the original list if some basic definitions of what the performance indices imply are discussed and accepted.

For example, should the park orbit time at Mars be taken as a specified constraint, then the injection and departure maneuvers may be computed as coplanar velocity adjustments. This is a valid assumption as now the departure date from Mars is fixed, and as a result, is a constant in the performance index evaluation. Therefore, a trajectory satisfying the performance index will automatically specify the departure asymptote and, together with the already defined approach asymptote, the park orbit orientation required at Mars may be established. As a result, it may be concluded that a coplanar $\Delta V_{♂}$ into and out of the park orbit at Mars is a valid variable for use in the performance index. Parameters (5), (6) and the orientation portion of (9) are eliminated from the study.

In summary then, $\Delta V_{D\oplus}$, $LD_{\oplus}$, T_1, $\Delta V_{A♂}$, $\Delta V_{D♂}$, T_2, T_3, $\Delta V_{♀}$, T, and $\Delta V_{A\oplus}$ are the remaining parameters available for describing the results of the study. Notice that not all these parameters are independent. The independent parameters or variables available for manipulation are:

1. $LD_{\oplus}$ Launch date when mission is initiated.
2. T_1 Flight time on first leg of mission.
3. T Stay time in Martian park orbit.
4. T_2 Flight time on second leg of mission.
5. T_3 Flight time on third leg of mission.

Only power assisted Venus swing-by missions with dwell times at Mars will be studied. Zero dwell time at Mars corresponds to a power assisted swing-by of that planet.

The constraints on the study and resulting set of independent variables leads to a partitioned study; that is, the overall study will be broken into two separate studies. Study number one will be characterized by the independent variables $LD_{\oplus}$ and T_1. The second study will be investigated via the variables $T_{♂}$, T_2, and T_3 for a given Mars arrival date. Mars arrival date being one of the parameters described in the results of study number one.

The principal effort will be in the area of study number (2); that is, a powered flyby of Venus. The variables T_2 and T_3 will be varied as a function of the parameter $LD_{♂}$ (i.e., departure date from Mars).

$$LD_{♂} = AD_{♂} + T$$

where

$AD_{♂}$ = arrival date at Mars.

Study number (1) has been investigated in great detail by numerous individuals. Philco has also done an appreciable amount of work in this area. Results already generated from this phase of the study will be used in conjunction with those to be generated from the present study to display in graphical form, the overall mission savings in total ΔV required. Final conclusions will be drawn from a direct analogy of the powered flyby with the unpowered or gravity assisted flyby. Presentation of the results will be compatible with the presently accepted format for gravity assisted flybys. That is, the results, if possible, will be presented as an extension of the conclusions for unpowered flybys.

SECTION 4

PROGRAM DESCRIPTION AND CAPABILITIES

In order to obtain the parametric data required for analysis of the problem, it becomes necessary to construct a program capable of rapid analysis of swing-by missions. As a result, a FORTRAN IV Program was written for this explicit purpose and its output combined with the Stromberg-Carlson 4020 plotter. The added capability of automatic plotting further increased the rapidity with which useful swing-by mission data could be generated and analyzed. Figure 1 is an information flow diagram of the basic program. The program is constructed to utilize flight time from the launch to the target body as the independent parameter and flight time from the launch to the intermediate body as a secondary parameter. The role of these two variables may be reversed in plotting the output. As can be seen from the flow diagram, the program is capable of producing either parametric data or data associated with the minimization of zero search of a given function of flight time from launch to target body. To date the only function minimized has been the thrust augmented maneuver by Venus. More will be said of this later. It is planned in future weeks to try to minimize weight required on orbit at Mars and compare the characteristics of these solutions to those for minimizing the thrust-augmented maneuver by Venus. I_{SP} of the engine will be a parameter of the study.

The mathematical model defining the system assumes massless planets when computing the heliocentric portion of the trajectories. As a result, the conic sections over heliocentric space are defined by the radius vectors to both the launch and target bodies and the desired flight time between them. The exact procedure for computing the necessary conic section, under these assumptions, is described in Appendix A. A full patched conic solution of the problem was not deemed necessary for two basic reasons:

(1) as this is a feasibility study, the added EDPM computation time required for computing the patched conditions at both ends of the heliocentric conic was not justifiable; (2) the error introduced into the computation is of second order importance. Planetocentric conics were computed using a sphere of influence technique. That is, the difference vector between the heliocentric arrival, or departure, velocity vector of the probe and the planets velocity vector was assumed to define the hyperbolic energy at planet encounter.

The quantity, defined C_3 in astrodynamics jargon, and the definition of the patch distance permits computation of the probe velocity relative to the planet at the patch point. The actual computation required for solution of the incremental velocity correction at the swing-by planet is discussed in Appendix E. It should be noted here that all maneuvers required at the swing-by planet were defined to occur at departure from the sphere of influence of that planet. The required maneuvers are computed as coplanar maneuvers where the plane is defined by the arrival and departure asymptotic velocity vectors. This is not the optimum plane for application of the required velocity maneuver in all cases. It is the optimim point of application for a maneuver whose purpose it is to only rotate the asymptote. Under the original asymptions of the study (i.e., only maneuvers of the magnitude of nominal midcourse correction would be considered) the point of application of the corrective maneuver was at first assumed to be of secondary importance. After reading the work of Gobetz and Hollister and Prussing[5] it was decided to conduct an analysis into the sensitivity of point of application of the corrective maneuver to the desired alteration of the departure asymptotic velocity vector. Two sensitivity coefficients, as a function of point of application along the hyperbola were investigated: (1) rotation of the departure asymptote and (2) hyperbolic energy increase only. The goal of this investigation was to determine the magnification factor or error incurred in the solution of the ΔV_{φ} maneuver required by assuming it occurred at departure from the sphere of influence. To date, the study has not been completed but is in its final stages of program checkout.

Appendix D provides a detailed description of the analysis.

The Swing-By Mission Analysis Program was written so that the characteristics of an entry vehicle, using aerodynamic braking, could be input to the program. The vehicle was assumed to be defined in terms of an entry weight-entry velocity curve (a quadratic). The particular vehicle assumed in the current study was one of a bi-conic configuration. Its entry weight vs. entry velocity characteristics are shown in one of the SC-4020 outputs contained in Appendix F.

Included in the output is the on-orbit weight required for completion of the mission. This weight is calculated as a function of the flight time from launch to target planet, flight time from target to intermediate body and defined entry vehicle as well as the I_{SP} of the injection and maneuvering engines. The existing version of the program assumes all tankage and associated structure to be carried along in the mission until atmospheric entry at Earth return.

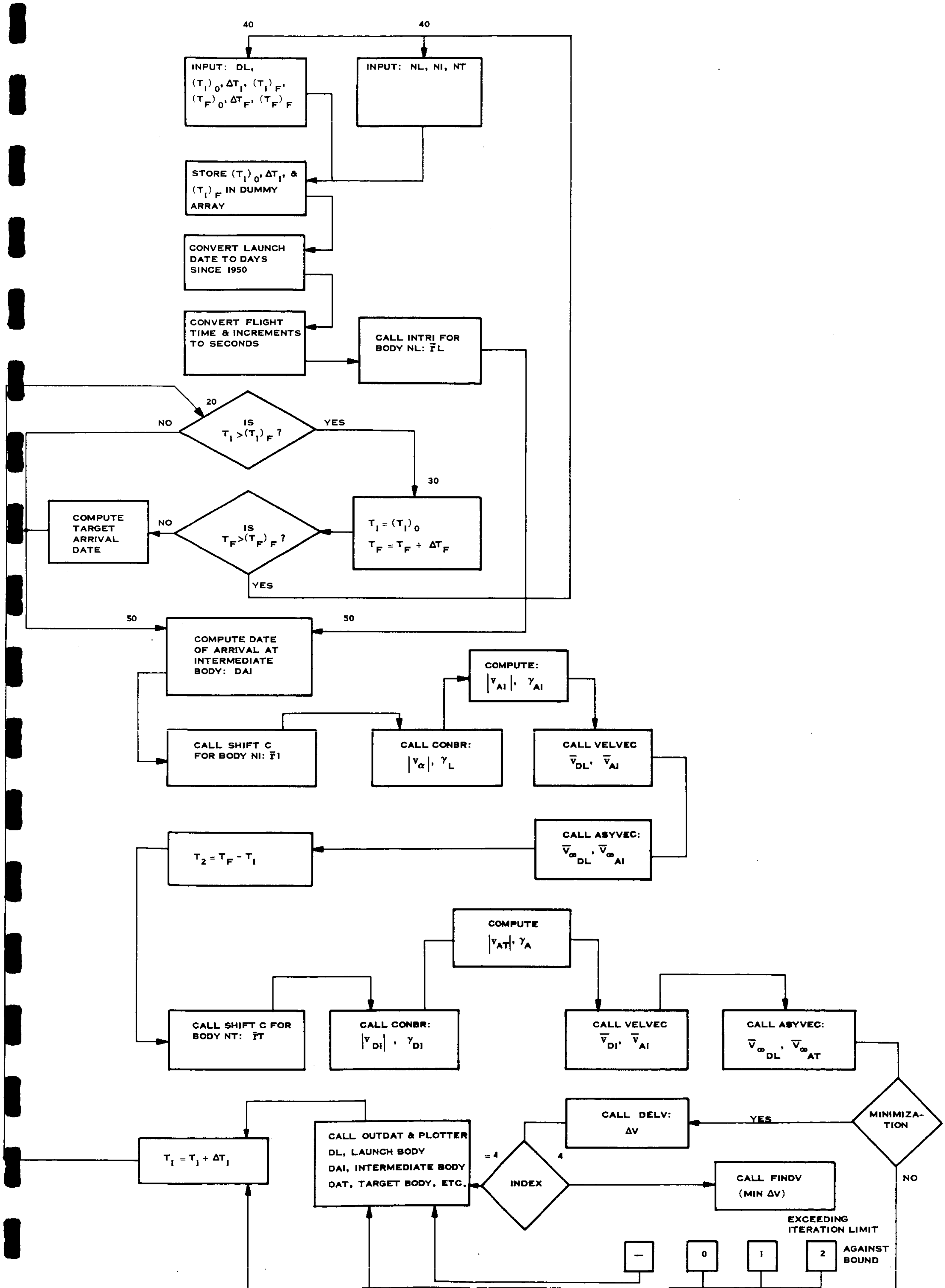
Due to the possibility of performing thrusting maneuvers at the intermediate planet of sizeable magnitude, the existing program will be altered to include the minimum ΔV computation on the flyby maneuver (i.e., should be the sensitivity coefficient study justify this). The logic presented by Hollister and Prussing will be utilized for selection of the near-optimum point of application. The reasoning behind wanting to perform maneuvers on the order of one or two kilometers per second will be synopsized in the section entitled "Future Work". Appendix C provides a detailed explanation of the procedure for computing the ΔV required at the optimum point of application.

4.1 PARAMETRIC ANALYSIS

Figures 2-A and 2-B are examples of the input required for a parametric analysis of a flyby mission and the corresponding output for a typical flyby mission. The input, as this is a FORTRAN IV Program, is via the namelist statement. A definition of the input quantities is as follows:

PROGRAM FLOW DIAGRAM

Figure 1



DL1 = year.month and day of launch.

DL2 = hour.minute and second of launch.

T10 = initial first leg flight time.(days).

DT1 = increment of first leg flight time.(days).

T1F = final first leg flight time.(days).

TFO = initial total flight time.(days).

DTF = incremental total flight time.(days).

TFF = final total flight time.(days).

NL = launch body numer.

NI = intermediate body number.

NT = target body number.

XIL2I = initialization vector (all zero).

XII2T = initialization vector (all zero).

IV = initialization vector (0, 1, 0).

TIM = period for which conic solution is to be applied to planetary ephemeris.

INDEX = key for selection of function to be optimized.

INDEX = 1 = maneuver by Venus.

INDEX = 2 = maneuver by Venus + injection velocity at Mars.

INDEX = 3 = maneuver by Venus + injection velocity at Mars + entry velocity at Earth.

INDEX = 4 = injection velocity at Mars.

INDEX = 5 = entry velocity at Earth.

INDEX = 6 = parameteric analysis (no optimization).

NOPT = $\begin{cases} 1 = & \text{output of intermediate data during optimization.} \\ 0 = & \text{no output of intermediate data during optimization.} \end{cases}$

NPLOT = $\begin{cases} 1 = & \text{plotting of data.} \\ 0 = & \text{no plotting of data.} \end{cases}$

NFINAL = $\begin{cases} 1 = & \text{final case.} \\ 0 = & \text{not final case.} \end{cases}$

MASS = $\begin{cases} 1 = & \text{mass calculations .} \\ 0 = & \text{no mass calculations .} \end{cases}$

```
$IN
BL1     =    0.78030200E 04,
BL2     =    0.12000000E 04,
TI0     =    0.16999999E 05,
DT1     =    0.20000000E 03,
TIF     =    0.22000000E 05,
TFO     =    0.23999999E 05,
DTF     =    0.20000000E 03,
TFF     =    0.27000000E 05,
NL      =           4,
NI      =           3,
NT      =           0,
XIL2I   =   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,
            -0.00000000E-19,
XII2T   =   -0.C0000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,
            -0.00000000E-19,
IV      =           0,            1,             0,
TIM     =    0.20000000E 02,    0.20000000E 02,    0.20000000E 02,    0.20000000E 02,    0.20000000E 02,
             0.20000000E 02,
INDEX   =           6,
NOPT    =           0,
NPLOT   =           1,
NFINAL  =           1,
MASS    =           1,
TX      =    0.24999999E 05,    0.49999999E 05,    0.64999999E 05,
FTX     =    0.95000000E 04,    0.11200000E 05,    0.13999999E 05,
```

Input Required for Parametric Analysis

Figure 2-A

TX = three values of entry velocity associated with entry vehicle selected.

FTX = three values of entry weight associated with vehicle selected.

These inputs are identical for either a parametric analysis or an optimization run except for the selection of the appropriate index value.

Figure 2-B contains an example case of the output of the program. A line-by-line description of the output is as follows:

Line 1

L.D. = year.month and day of launch.

DELV* = injection velocity required from 100 Km circular park orbit. (Km/sec).

C3 = injection energy (i.e., V_∞^2) (Km^2/sec^2).

I.D. = year.month and day of arrival at intermediate body.

DELV** = corrective maneuver required at Venus ($^m/s$).

RP = radius of closest approach (Km).

C3A = arrival asymptotic velocity (intermediate body)-(Km/sec)**2.

C3D = departure asymptotic velocity (Km/sec)**2.

A.D. = arrival date at target body (year.month and day).

DELV*** = retro required to arrive on a circular park orbit of 184 Km on Earth arrival (Km/sec).

AV = asymptote velocity on arrival at target body. (Km/sec).

TF = total flight time (days).

T1 = first leg flight time (days).

PSI = thrusting angle required referenced to departure velocity vector at maneuver at swing-by planet (degree).

Line 2

The hour, minute/and seconds of the corresponding date shown on line 1.

Line 3

PSI1	=	central angle traveled during the heliocentric phase of the flight from launch to intermediate body (degree).
PSI2	=	central angle traveled during the heliocentric phase of the flight from intermediate body to target body (degree).

Line 4

Entry Velocity	=	velocity at perigee of the approach hyperbola to Earth (a 184 Km perigee) Km/sec.
Entry Weight	=	structure + heatshield at atmospheric entry to Earth (#'s).
Weight Out of orbit	=	weight injected into a heliocentric conic from the launch planet. (#'s).
Park Orbit Weight	=	weight on park orbit at launch planet required to deliver study weight at Earth (#'s).

The program generated to compute these quantities as well as others is extremely fast in operation. On the average, the program requires 1.2 seconds to compute a solution to the swing-by problem, write the solution on an output tape in the format shown and write the corresponding data on a plot tape.

```
$ END

   CASE  13  SWING BY SEARCH  1978        DEERWESTER DATA
          VENUS SWING-BY MISSION , RETURN FROM MARS
          *    LAUNCH FROM 100 KM CIRCULAR PARK ORBIT (COPLANAR)
          **   INPLANE DELTA V REQUIRED AT VENUS (NO TARGETING)  MIN DV
          ***  DELTA V REQUIRED TO ENTER A 100 KM CIRC PARK ORBIT AT TRG

  L.DJ    DELV * C3        I.D.     DELV **       RP       C3A      C3D      A.D.     DELV ***    AV       TF        TI        PSI
 7803.02  5.11  49.70   7808.19   2220.21 30540.78    58.71     97.97  7810.28   -10.27     14.32   240.00    170.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  138.25   PSI2=  117.25
 ENTRY VEL=   18.11   ENTRY WT=  12802.25   WEIGHT OUT OF ORBIT=  24446.13   PARK ORBIT WT= 108335.63
 7803.02  5.09  49.38   7808.21   2546.98 26834.60    54.90     99.49  7810.28   -10.53     14.65   240.00    172.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  141.41   PSI2=  114.09
 ENTRY VEL=   18.38   ENTRY WT=  12976.75   WEIGHT OUT OF ORBIT=  27254.43   PARK ORBIT WT= 120130.30
 7803.02  5.07  49.02   7808.23   2824.68 23628.94    52.04    101.18  7810.28   -10.81     14.99   240.00    174.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  144.58   PSI2=  110.92
 ENTRY VEL=   18.65   ENTRY WT=  13162.08   WEIGHT OUT OF ORBIT=  29973.18   PARK ORBIT WT= 131301.71
 7803.02  5.05  48.61   7808.25   3053.08 20815.38    50.09    103.08  7810.28   -11.09     15.35   240.00    176.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  147.74   PSI2=  107.76
 ENTRY VEL=   18.93   ENTRY WT=  13359.33   WEIGHT OUT OF ORBIT=  32515.60   PARK ORBIT WT= 141447.39
 7803.02  5.02  48.15   7808.27   3232.39 18312.53    49.01    105.20  7810.28   -11.39     15.71   240.00    178.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  150.90   PSI2=  104.60
 ENTRY VEL=   19.23   ENTRY WT=  13569.80   WEIGHT OUT OF ORBIT=  34799.27   PARK ORBIT WT= 150205.33
 7803.02  4.99  47.65   7808.29   3362.79 16061.65    48.81    107.61  7810.28   -11.69     16.08   240.00    180.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  154.06   PSI2=  101.44
 ENTRY VEL=   19.54   ENTRY WT=  13795.07   WEIGHT OUT OF ORBIT=  36746.81   PARK ORBIT WT= 157267.80
 7803.02  4.96  47.13   7808.31   3443.12 14014.70    49.52    110.34  7810.28   -12.02     16.47   240.00    182.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  157.22   PSI2=   98.28
 ENTRY VEL=   19.86   ENTRY WT=  14036.98   WEIGHT OUT OF ORBIT=  38276.56   PARK ORBIT WT= 162357.68
 7803.02  4.95  46.94   7809.02   3461.37 12001.79    51.37    113.48  7810.28   -12.35     16.88   240.00    184.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  160.38   PSI2=   95.12
 ENTRY VEL=   20.20   ENTRY WT=  14297.76   WEIGHT OUT OF ORBIT=  39195.62   PARK ORBIT WT= 165707.21
 7803.02  4.93  46.67   7809.04   3359.67 10211.57    54.59    116.00  7810.28   -12.66     17.25   240.00    186.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  163.53   PSI2=   91.96
 ENTRY VEL=   20.51   ENTRY WT=  14541.29   WEIGHT OUT OF ORBIT=  38699.33   PARK ORBIT WT= 162874.19
 7803.02  4.94  46.73   7809.06   3177.79  8366.77    60.05    119.82  7810.28   -13.02     17.67   240.00    188.00    180.00
 1200.00               1200.00                                         1200.00
 PSI1=  166.68   PSI2=   88.80
 ENTRY VEL=   20.86   ENTRY WT=  14833.30   WEIGHT OUT OF ORBIT=  37439.08   PARK ORBIT WT= 157721.69
 7803.02  4.99  47.46   7809.08   2759.35  6348.45    70.09    124.26  7810.28   -13.40     18.11   240.00    190.00    180.00
```

Data Output for Parametric Analysis

Figure 2-B

4.2 OPTIMIZATION CAPABILITIES

The swing-by program in its present version possesses the capability of minimizing six basic functions of the velocity maneuvers required to complete the Mars-Venus-Earth leg of the interplanetary mission. These functions were summarized and explained in the preceding section. In addition, the capability for minimizing on orbit weight at Mars is being added to the program. The input quantities required for a minimization run are identical to those described in the preceding section, except for the selection of the appropriate index values. The output quantities, their format, and the binary data tape written for SC-4020 Plotter Output, are also identical to those described earlier.

Figures 3-A and 3-B are typical examples of the input required and output data generated for a minimization run in which the ΔV maneuver at Venus was minimized.

The program has been written in such a way that minimization of any quantity expressed as a function of flight time from the launch to the target is possible.

```
$IN
DL1    =  0.78030200E 04,
DL2    =  0.12000000E 04,
TIO    =  0.16999999E 05,
DT1    =  0.20000000E 03,
T1F    =  0.22000000E 05,
TFO    =  0.23999999E 05,
DTF    =  0.20000000E 03,
TFF    =  0.27000000E 05,
NL     =           4,
NI     =           3,
NT     =           0,
XIL2I  = -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,
         -0.00000000E-19,
XII2T  = -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,
         -0.00000000E-19,
IV     =           0,        1,          0,
TIM    =  0.20000000E 02,    0.20000000E 02,    0.20000000E 02,    0.20000000E 02,    0.20000000E 02,
          0.20000000E 02,
INDEX  =           1,
NOPT   =           0,
NPLOT  =           0,
NFINAL =           1,
MASS   =           1,
TX     =  0.24999999E 05,    0.49999999E 05,    0.64999999E 05,
FTX    =  0.95000000E 04,    0.11200000E 05,    0.13999999E 05,
```

Input Required for Optimization Run (i.e. Minimization of ΔV Maneuver at Venus)

Figure 3-A

```
CASE  13  SWING BY SEARCH  1978         DEERWESTER DATA
      VENUS SWING-BY MISSION , RETURN FROM MARS
      *    LAUNCH FROM 100 KM CIRCULAR PARK ORBIT (COPLANAR)
      **   INPLANE DELTA V REQUIRED AT VENUS (NO TARGETING)  MIN DV
      ***  DELTA V REQUIRED TO ENTER A 100 KM CIRC PARK ORBIT AT TRG

  L.D.   DELV * C3       I.D.     DELV **     RP       C3A      C3D      A.D.    DELV ***    AV       TF        T1       PSI
7803.02  5.07  49.08  7809.09  2493.96  6050.00   79.59   126.90  7810.28   -13.61     18.36   240.00   191.06   -18.54
1200.00               1327.49                                     1200.00
PSI1=  171.49   PSI2=   83.97
ENTRY VEL=   21.45   ENTRY WT=  15328.34   WEIGHT OUT OF ORBIT=  31699.88   PARK ORBIT WT= 139007.26
7803.02  4.86  45.47  7809.22  5033.20  6050.00   92.13   187.98  7810.28   -16.84     22.05   240.00   203.85   -28.98
1200.00               821.00                                      1200.00
PSI1=  191.84   PSI2=   63.77
ENTRY VEL=   24.68   ENTRY WT=  18453.18   WEIGHT OUT OF ORBIT=  79969.49   PARK ORBIT WT= 329608.30

7803.02  5.06  48.81  7809.09  1825.94  6050.00   77.98   110.73  7810.30   -12.76     17.36   242.00   190.91   -20.93
1200.00               955.02                                      1200.00
PSI1=  171.26   PSI2=   86.20
ENTRY VEL=   20.60   ENTRY WT=  14620.66   WEIGHT OUT OF ORBIT=  24888.82   PARK ORBIT WT= 108651.10
7803.02  4.78  44.10  7809.23  4071.92  6050.00   91.71   164.52  7810.30   -15.87     20.95   242.00   204.73   -31.55
1200.00               530.11                                      1200.00
PSI1=  193.23   PSI2=   64.37
ENTRY VEL=   23.71   ENTRY WT=  17440.73   WEIGHT OUT OF ORBIT=  57119.92   PARK ORBIT WT= 229891.89

7803.02  5.04  48.49  7809.09  1222.02  6050.00   75.94    96.88  7811.01   -11.98     16.43   244.00   190.71   -22.06
1200.00               503.38                                      1200.00
PSI1=  170.94   PSI2=   88.51
ENTRY VEL=   19.82   ENTRY WT=  14008.26   WEIGHT OUT OF ORBIT=  19998.96   PARK ORBIT WT=  86829.83
7803.02  4.70  42.73  7809.24  3200.89  6050.00   92.33   145.70  7811.01   -15.02     19.99   244.00   205.84   -34.97
1200.00               814.16                                      1200.00
PSI1=  194.99   PSI2=   64.61
ENTRY VEL=   22.86   ENTRY WT=  16609.85   WEIGHT OUT OF ORBIT=  42206.19   PARK ORBIT WT= 165804.07

7803.02  4.60  41.24  7809.25  2410.12  6050.00   94.50   131.63  7811.03   -14.32     19.19   246.00   207.37   -39.44
1200.00               2057.40                                     1200.00
PSI1=  197.40   PSI2=   64.19
ENTRY VEL=   22.16   ENTRY WT=  15959.49   WEIGHT OUT OF ORBIT=  32208.69   PARK ORBIT WT= 123207.00

7803.02  5.01  47.93  7809.08   200.49  6050.00   72.11    75.27  7811.05   -10.63     14.77   248.00   190.27   -24.30
1200.00               1829.59                                     1200.00
PSI1=  170.25   PSI2=   93.21
ENTRY VEL=   18.47   ENTRY WT=  13039.95   WEIGHT OUT OF ORBIT=  13824.32   PARK ORBIT WT=  59451.33
```

Data Generated for Optimization Run

Figure 3-B

SECTION 5

1978 OPPORTUNITY

The 1978 opportunity was chosen as the initial area for the investigation of thrust augmented maneuvers. This opportunity was selected for two basic reasons:

1. Investigations by SOHN[1] have shown this to be a marginal opportunity in terms of characteristic velocity required when compared to a direct return.

2. The 1978 opportunity is probably representative of the initial opportunity for a manned flyby of Venus on the return from Mars. The fact that the 1978 opportunity is marginal as far as feasibility is concerned makes it most attractive for analyzing in terms of thrust augmented maneuvers at Venus.

The benefits of using thrust augmented maneuvers by Venus, should they be acceptable, would be the widening of the permissible launch window for a given boost vehicle configuration. The launch window here is meant to imply the permissible date of launch out of Mars park orbit. An alternate derivative would hopefully be an appreciable shortening of total flight time on the return leg or an appreciable reduction in atmospheric entry velocity on Earth return. The latter could possibly lead to a reduction in the heat shield weight required, and thereby a reduction of the weight required on orbit at Mars. Hidden in these suppositions is the fact that any maneuver performed on the Venus flyby will require engine, propellant, and structure to be boosted out of the park orbit. In essence, any weight savings accrued on Earth entry, due to applying a thrusting maneuver at Venus, must be traded off with the weight penalty associated with making that maneuver by Venus. These considerations as well as a parametric comparison to a gravity assisted flyby have been applied to the 1978 mission.

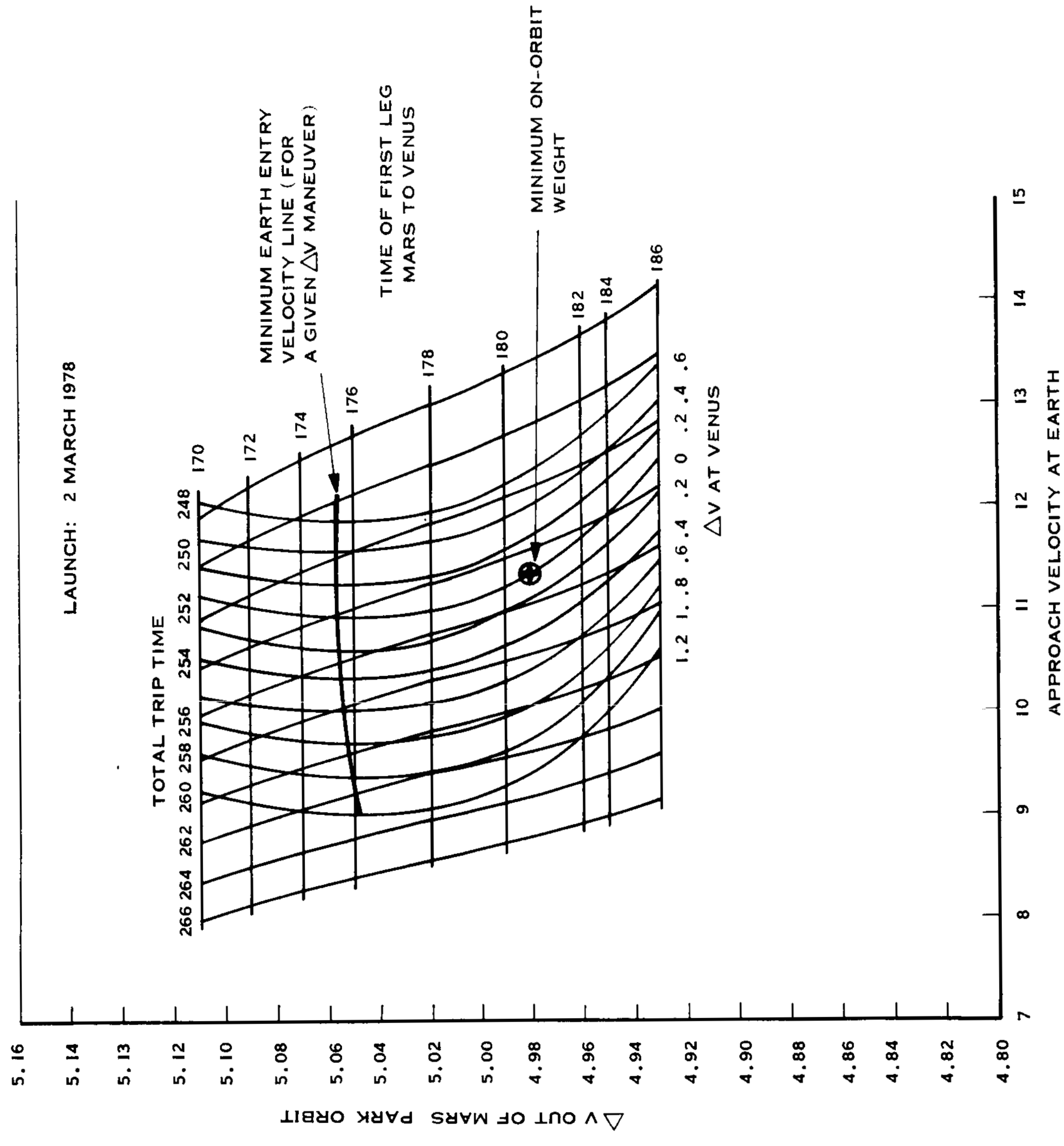
5.1 PARAMETRIC ANALYSIS RESULTS

Parametric data on the 1978 opportunity was generated for a Mars launch date of March 2, 1978. The data was generated in the form of Stromberg-Carlson 4020 plots. These plots are shown in Appendix F.

The data content of the computer runs was analyzed and reduced to a single plot - Figure 4. Plots such as those shown in Figure 4 could be generated for each launch date from Mars; however, only the March 2 date will be discussed in this report. Figure 4 contains all the information necessary for determining the merits of a power-assisted flyby as compared with gravity-assisted flybys. The abscissa contains the approach velocity to Earth, whereas the ordinant describes the velocity increment required for injection onto the departure hyperbola at Mars. A 100 kilometer aerocentric park-orbit is assumed to exist. The basic grid contains total flight time and flight time from Mars to Venus. Total flight time curves are those running nearly vertical on the page. First-leg flight times are those running horizontally across the page. The smooth arcs overlaying this grid represent constant incremental velocity maneuvers by Venus. Notice that contours of equal magnitude occur on either side of the zero curve. It was found that these contours belong to Type I, Class I and II trajectories, respectively. A quick review of the parameters displayed in Figure 4 will reveal a basic incompatibility; that is, it is impossible to "improve" all parameters through the application of a thrusting maneuver at Venus. From the aspect of reducing mission characteristic velocity, Figure 4 indicates that one should proceed toward the origin of the plot; that is, find the most favorable tradeoff between incremental velocity applied and the associated reduction in launch velocity required, and entry velocity dictated. However, following this philosophy causes total trip time to increase - a derivative of applying an incremental velocity not considered beneficial. Total trip time on the first leg is also seen to increase. If one is willing to accept an increase in total trip time, then it becomes possible to trade off the reduction in entry velocity, entry weight, and consequently payload on orbit, against the incremental velocity required.

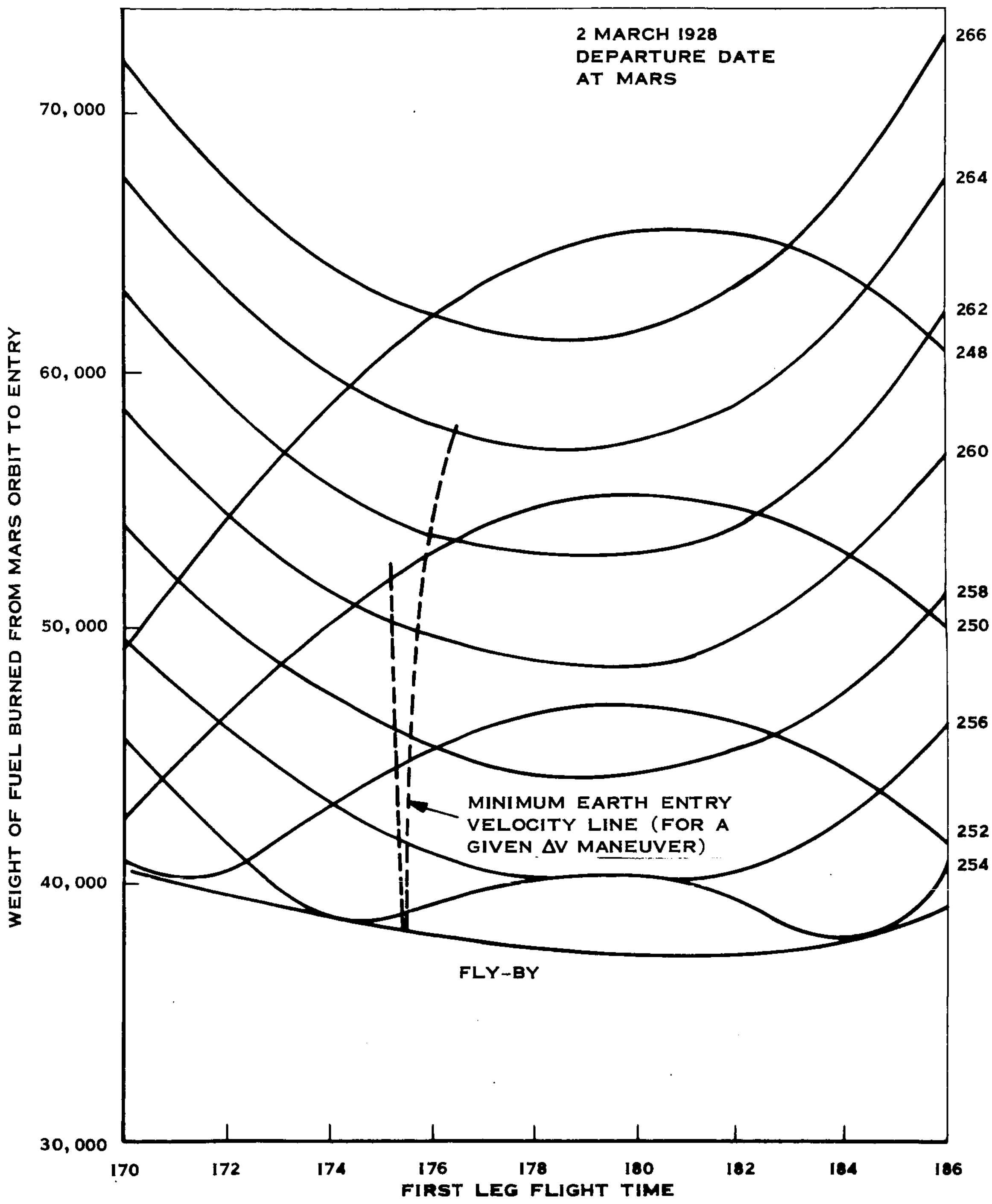
An interesting aspect of Figure 4 is the relative insensitivity of minimum Earth entry velocity,for a given incremental velocity, to the total trip time. That is, the minimum Earth entry velocity, regardless of the total trip time and incremental velocity assumed, remained fixed at approximately 175 to 176 days flight time on the first leg. This conclusion is, of course, subject to a March 2, 1978, launch and the range of parameters shown in Figure 4. Figure 4 indicates that a net reduction in mission characteristic velocity of approximately .8 K/S is possible for the range of parameters considered. In particular, by working along the minimum Earth entry line an incremental velocity by Venus of 1.2 K/S is seen to reduce Earth approach velocity by approximately 2 K/S. This is, as stated, a net reduction of approximately .8 K/S in characteristic velocity. The velocity increment needed for injection onto the departure hyperbola at Mars is seen to remain nearly a constant. Trip time for this velocity tradeoff increases by 10 days - approximately 4%. At first glance the tradeoff seems to be a favorable one. However, when the added cost, in terms of propellant, structure, reliability, etc., of performing the incremental velocity change at Venus is considered, as well as the human factors associated with longer times in space, the complexion of the picture changes completely.

Figure 5 contains a description of amount of fuel required on orbit at Mars to complete the mission. The propellant weight shown takes into account the weight of the bi-conic entry vehicle needed for a mission of the duration shown (i.e., total flight time) and the dictated incremental velocity by Venus. The weight characteristics of the bi-conic entry vehicle as a function of Earth entry velocity are contained in the SC-4020 output presented in Appendix F. A quick glance at Figure 5 reveals that the envelope skirting the bottom of the constant flight time contours is that associated with the pure flyby maneuver. The .8 K/S reduction in Earth approach velocity translates into an actual reduction of useful payload on orbit at Mars; or, alternately, the capability for applying a 1.2 K/S maneuver at Venus costs approximately 20,000 lbs. of propellant on orbit at Mars. All propellant weights are calculated assuming a liquid hydrogen



Power Assist Comparison (Characteristic Velocity Basis)

Figure 4



Power Assist Comparison
(Fuel Required on Orbit Basis)

Figure 5

engine having a specific impulse of 350 seconds. The dashed vertical line in Figure 5 is the same minimum Earth entry velocity line described in Figure 4. It is interesting to note that minimum payload on orbit occurs for a total trip time somewhere between 254-256 days and the associated first leg trip time is approximately 180-181 days. By referring to Figure 4 again, and the zero incremental velocity line, it is seen that minimum on-orbit weight at Mars does not correspond to minimum Earth entry velocity and subsequently minimum Earth entry weight.

Subject to the constraints assumed in the study, entry vehicle definition, and I_{sp} of the engine assumed to perform all Δv maneuvers, power-assisted flybys of Venus do not seem feasible for this opportunity and launch date.

It should be mentioned that gravity-assisted flybys of Venus exist for a Type II trajectory to Venus; however, the entry velocities at Earth are on the order of 17-18 K/S - a velocity deemed not worthy of further investigation.

5.2 MINIMIZATION OF SWING-BY MANEUVERS

The conditions for which the parametric data was derived in the preceding section were input to the Swing-By Mission Analysis Program, and a minimization of the incremental velocity required by Venus was performed. Figures 6 through 9 contain the significant results of that minimization.

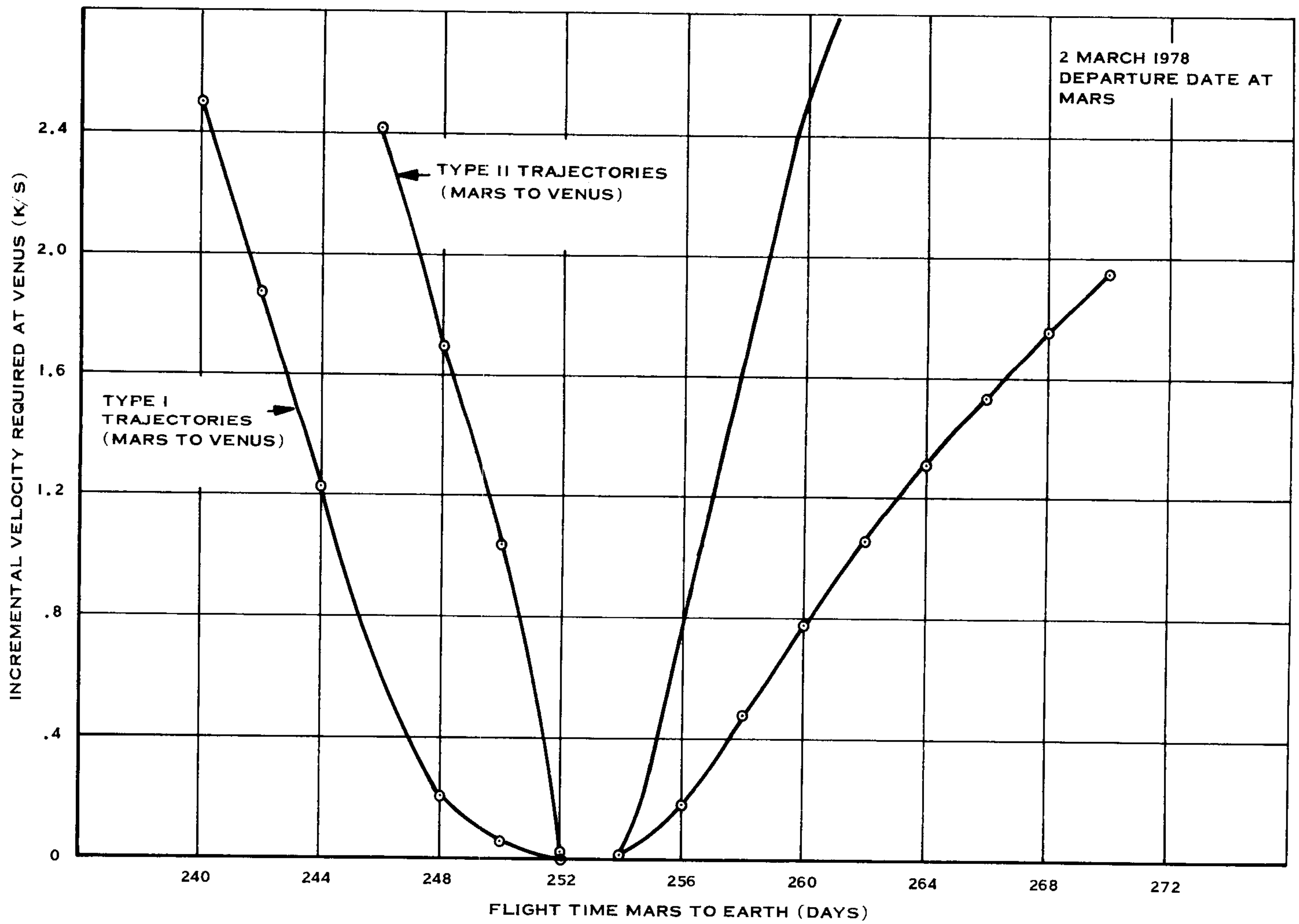
Figure 6 clearly illustrates the locus of points of two minimum incremental velocity mission possibilities. The steeper and more restricted set of opportunities (restricted in the sense of flight time from Mars to Venus) corresponds to Type II trajectories, whereas the remaining set of possibilities correspond to Type I trajectories. As the Type II trajectories have excessively high Earth entry velocities, they were not considered in the study. Plots such as these can be rapidly generated on the Swing-By Mission Analysis Program and can provide an assessment of the characteristics of the mission.

The total weight of the entry vehicle (i.e., structure and heat shield) required for successful completion of the minimum incremental velocity mission is contained in Figure 7. The net change between any two points on the curve is a direct reflection of the savings in cost of additional heat shield weight.

The apparent discontinuity in the curve, between 251 and 255 days flight time from Mars to Earth, occurs over the interval of unpowered flybys. The points contained in this figure are associated with the Type I trajectory.

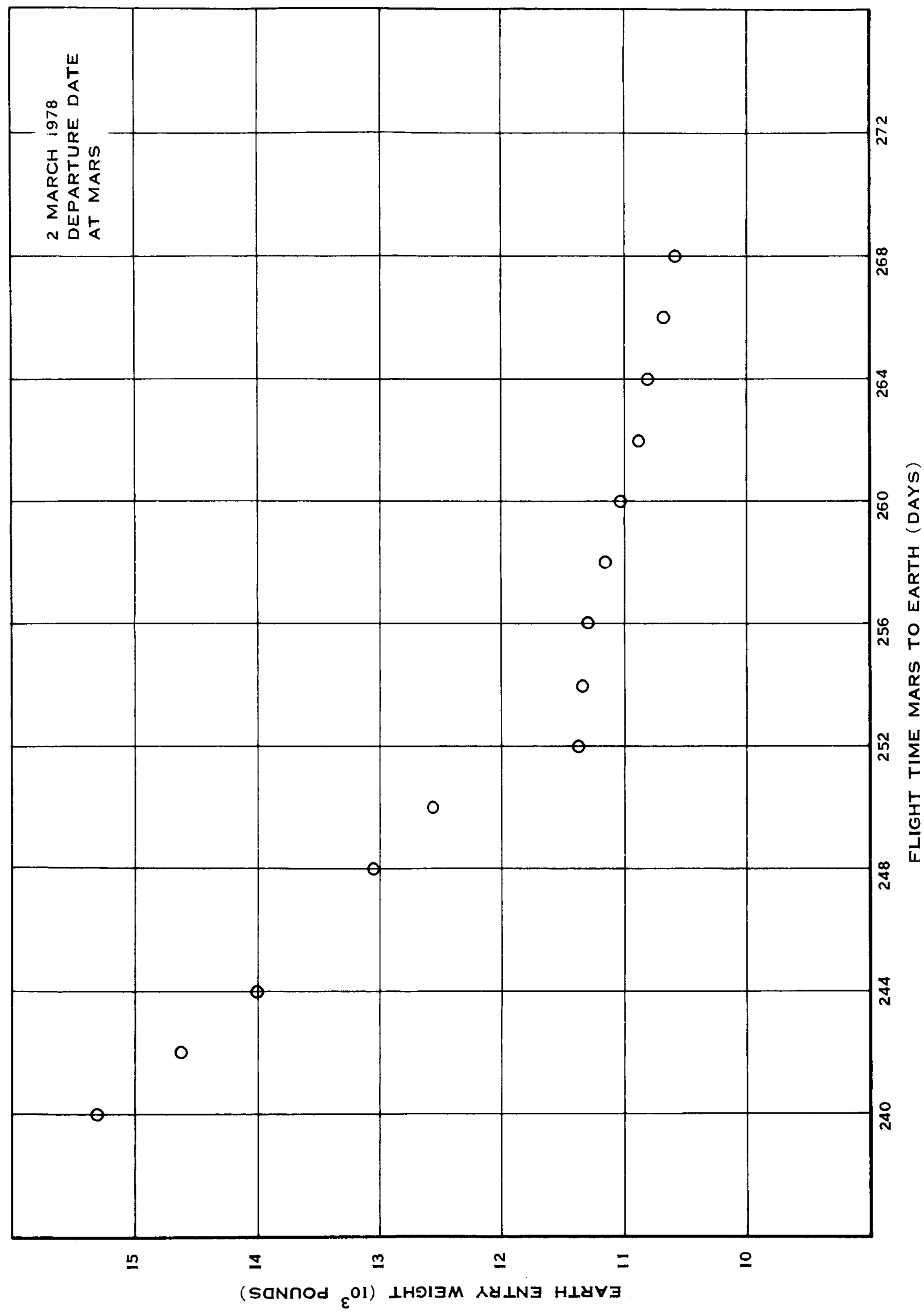
In a similar manner, Figures 8 and 9 describe the weight out of Mars park oribt and the weight required on Mars park orbit for minimum incremental velocity missions. These figures verify the conclusion reached earlier: minimum mission weight does not occur at the same conditions for which entry weight is a minimum.

Figures of the type shown provide a quick analysis of the mission characteristics associated with a given performance index. The test case described here served as both a check case for verification of program operation and a means for acquiring insight into the manner in which different parameters trade off on swing-by missions. However, only the performance index of incremental velocity required on the Venus swing-by mission has been investigated to date.



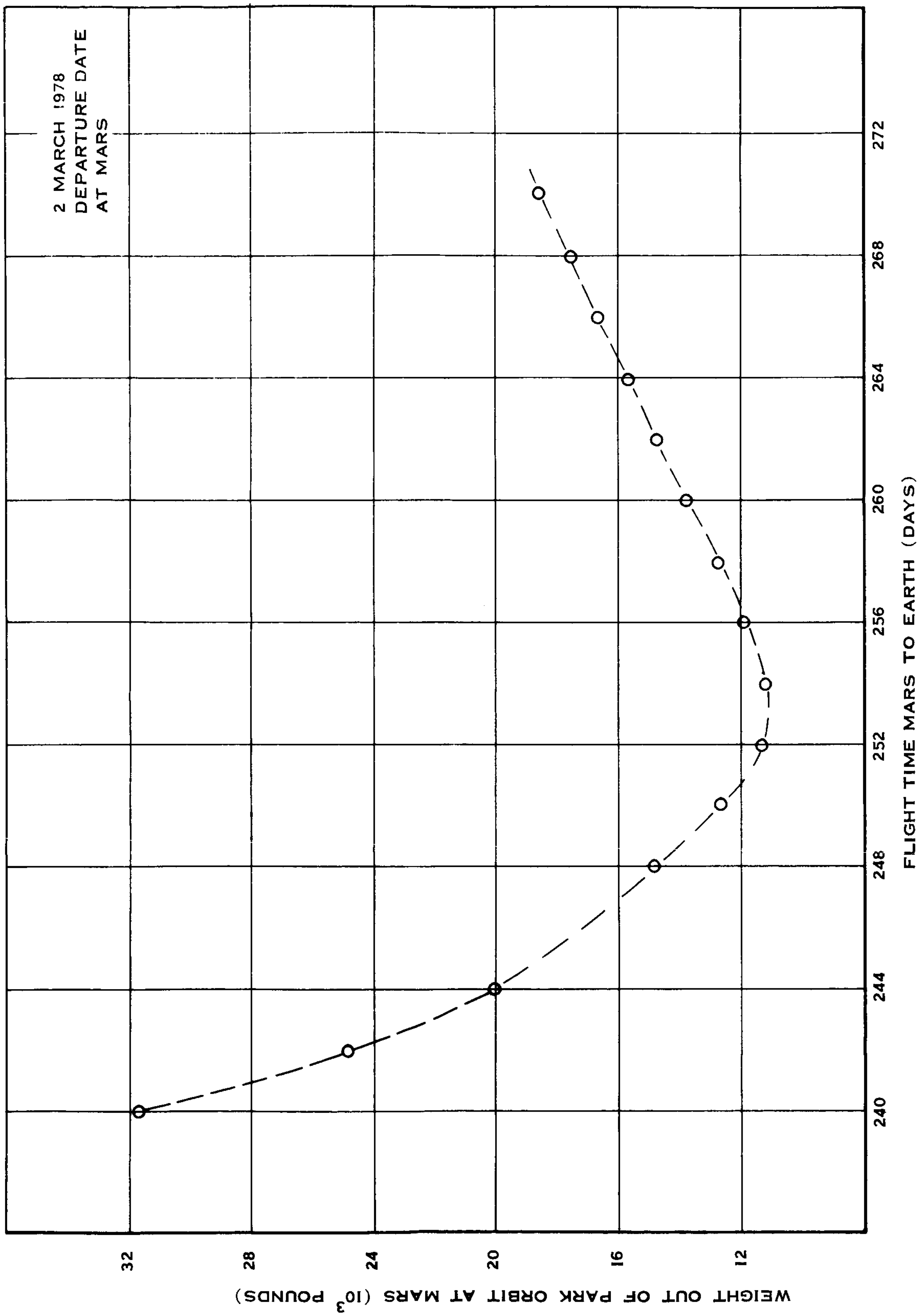
Minimum Incremental Velocity Trajectories for Mars Mission Return

Figure 6



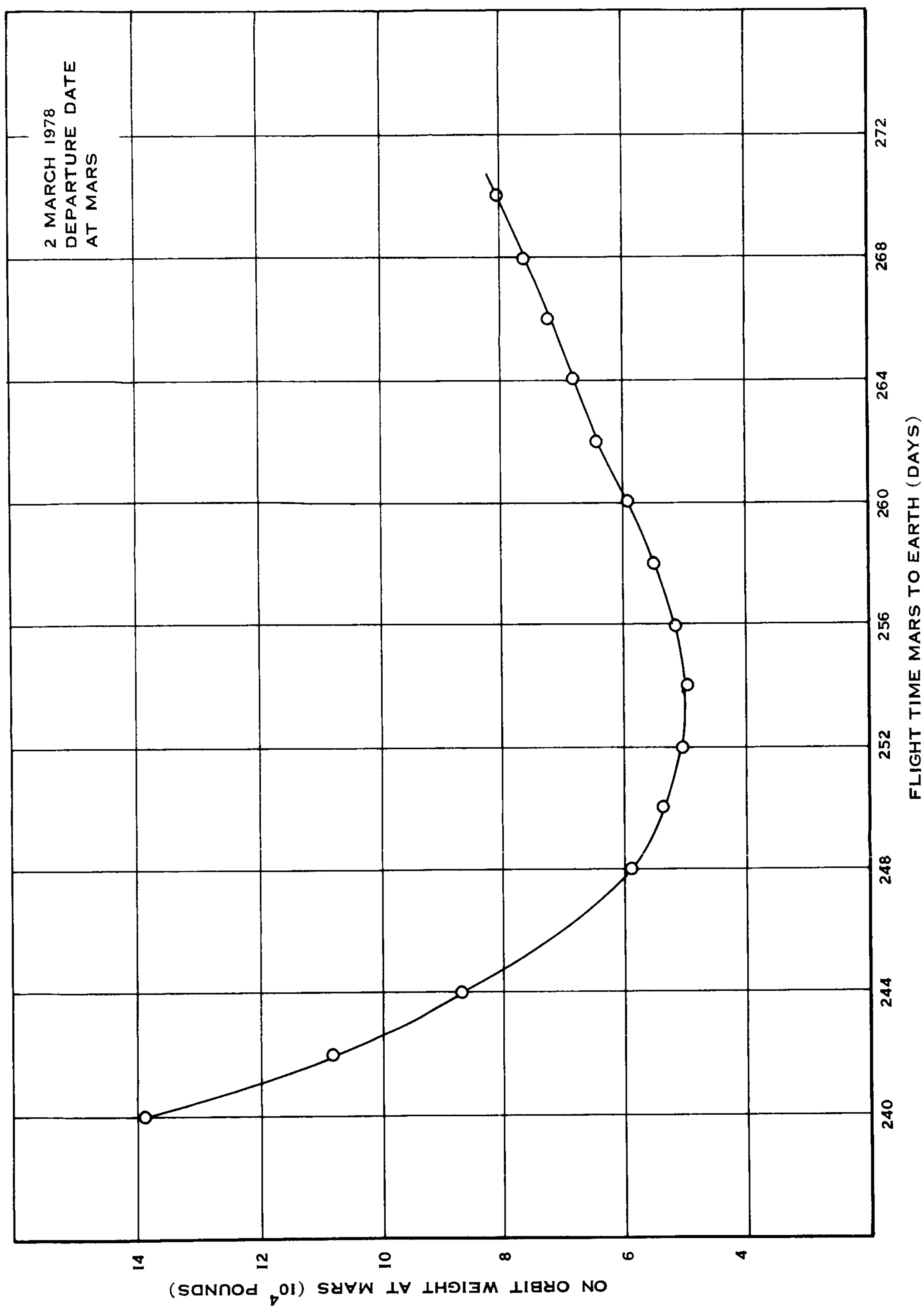
Vehicle Earth-Entry Weight For Minimum Incremental Velocity Missions

Figure 7



Weight Required on Departure From Mars For Minimum Incremental Velocity Missions

Figure 8



Weight Required on Orbit At Mars for Minimum Incremental Velocity Missions

Figure 9

SECTION 6

SUMMARY, CONCLUSIONS AND PHASE II WORK

The project achievements as a result of effort expended during the pre mid-term phase of the contract can be summarized as follows:

1. Generation of an operational program capable of analyzing swing-by missions either on a parametric basis or the optimizing of a stated performance index. The program obtains solutions at a rapid rate (i.e., a complete solution for a swing-by mission every 1.2 seconds).

2. Integrated into the basic program, the capability for using the Stromberg-Carlson plotter as an auxiliary output device. This capability permits the rapid generation and evaluation of swing-by mission data.

3. The plotted output has been arranged so that any two functions of total flight time or flight time on the first leg may be plotted against each other with either total flight time or first leg flight time serving as the secondary parameter on each graph (see Appendix F).

4. An investigation and derivation of the equations required to compute the incremental velocity necessary to transfer from one hyperbola onto a specified second hyperbola at any radius vector common to the two hyperbolas. The results of the derivation are identical to those obtained by Battin and used by Hollister and Prussing in Reference (5).

5. The theoretical derivation, programming, and checkout of a linear error propagation along a specified hyperbola with given end conditions on the linear equations at departure from the sphere

of influence. This study will determine the error introduced into the incremental velocity calculations by assuming that all maneuvers occur at the sphere of influence. Useful data has not yet been obtained from this program.

6. Experimentation into different ways of presenting the swing-by data so as to show most directly the benefits gained through a power-assisted maneuver culminated in a graphical display like that shown in Figure 4. This single plot permits the rapid evaluation of power-assisted maneuvers on the basis of a "pure" swing-by. As discussed earlier, any savings in mission characteristic velocity must be carefully traded off with the cost of performing the thrust-augmented maneuver. Cost here is meant to imply the increase or decrease in weight required on orbit at Mars in order to successfully complete the mission. These areas of consideration are reflected in the analysis of the test case shown.

7. It has been tentatively concluded that power-assisted swing-by maneuvers of the planet Venus during the return phase of a Mars mission are not feasible. This conclusion was reached for the 1978 opportunity and in particular for a launch from Mars on 2 March 1978. The basis for comparison here was the gravity-assisted swing-by.

8. Minimum mission weight does not occur at the same conditions for which minimum Earth entry weight occurs.

9. Swing-by missions flown in the 1978 time period, assuming a low approach velocity to Earth is of primary importance, are of Type I (i.e., the central angle on the Mars-Venus leg is less than 180 degrees).

The Phase II portion of the contract will emphasize the following areas:

1. Continue the investigation of the 1978 opportunity, for different launch dates from Mars, and verify the tentative conclusions drawn to date on power-assisted maneuvers.

2. Compare power-assisted maneuvers on the basis of direct return missions.

3. Analyze as many opportunities as time permits (opportunities occurring after 1978).

4. Generate a compact format, if both feasible and possible, for presentation of the merits of thrust-augmented maneuvers covering an entire opportunity.

5. Generate a compact format for tying together the outbound and return legs of an interplanetary mission.

As many of the above areas as possible will be covered in the time remaining.

SECTION 7

FUTURE WORK AREAS

Work areas worthy of further investigation might be summarized as:

1. Analyze the effects of using correction maneuvers by Venus where the thrust is provided by engines of higher I_{sp} than those used for injection onto park orbit at Mars (i.e., $I_{sp} > 350$).

2. Investigate the benefits derived from staging and using correction maneuvers on the order of midcourse corrections as compared to zero staging and using a restartable engine for a sizeable maneuver by Venus (i.e., trade off the effects of staging and I_{sp}).

3. Determine the savings in cost, as it effects mission time, weight, etc., of different types of entry vehicles (i.e, bi-conic, Apollo, etc.).

4. Analyze outbound trips to Mars as well as inbound trips from Mars for opportunities occurring during the 1980's (i.e., a continuation of the Phase II effort).

5. Determine the navigation and guidance requirements for a power-assisted swing-by mission should an opportunity occur where thrust-augmented maneuvers yield desirable changes in mission characteristics.

SECTION 8

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6. <u>Programming Manual for Quick-Look Mission Analysis Program</u>, Contract Number NAS5-3342, Goddard Space Flight Center, Greenbelt, Maryland, 24 January 1964.

APPENDIX A

CONIC SOLUTION GIVEN INITIAL AND TERMINAL RADIUS VECTORS AND THE TRANSIT TIME BETWEEN THEM

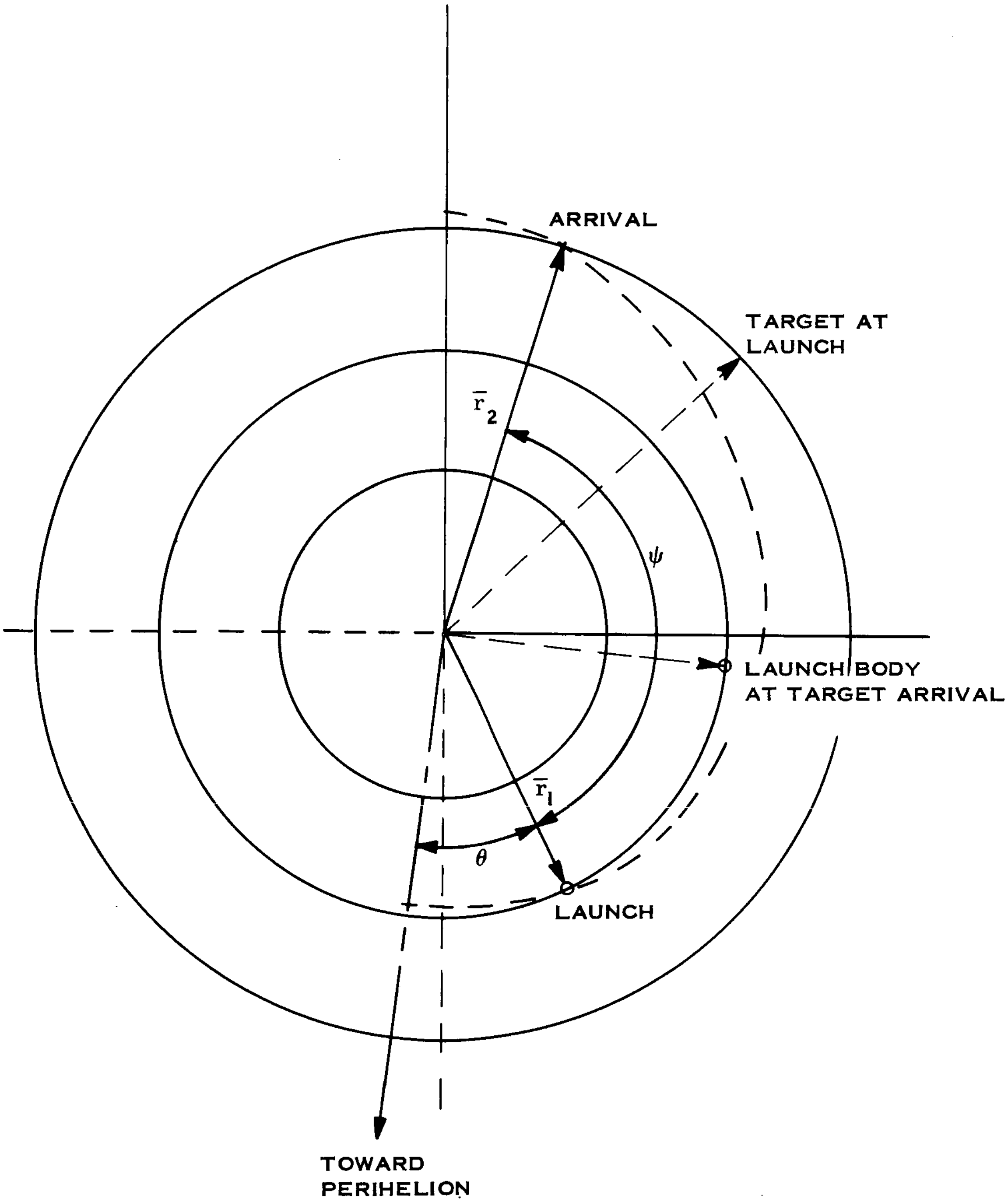
The swing-by program requires both the arrival heliocentric and depature heliocentric conics to be defined. Arrival and departure here is referenced to the intermediate or swing-by planet. Both the planet from which the mission originated (i.e., launch body) and the planet at which the mission terminated (i.e. target body) are considered to be massless. As a result, the heliocentric conic terminates at a point in space coincident with the required planet centers. As both the arrival date at the target planet is also known. Knowing the dates of arrival and departure permits, through the use of planetary ephemeris tables, the computation of the magnitude of the initial and final radius vectors.

Figure A-1 contains a description of the trajectory and the definition of symbols. As the terminal radii and elapsed time are known, the desired expressions may be derived as follows:

The polar equation of the conic is

$$r = \frac{p}{1 + e \cos \theta} \tag{A.1}$$

where r is the distance from the principal focus, p is the semi-latus rectum, e the eccentricity, and θ the true anomaly. In order that the launch body and the target body lies on the conic section, it is necessary that conic elements e, p, and θ are chosen such that



Trajectory Description

Figure A-1

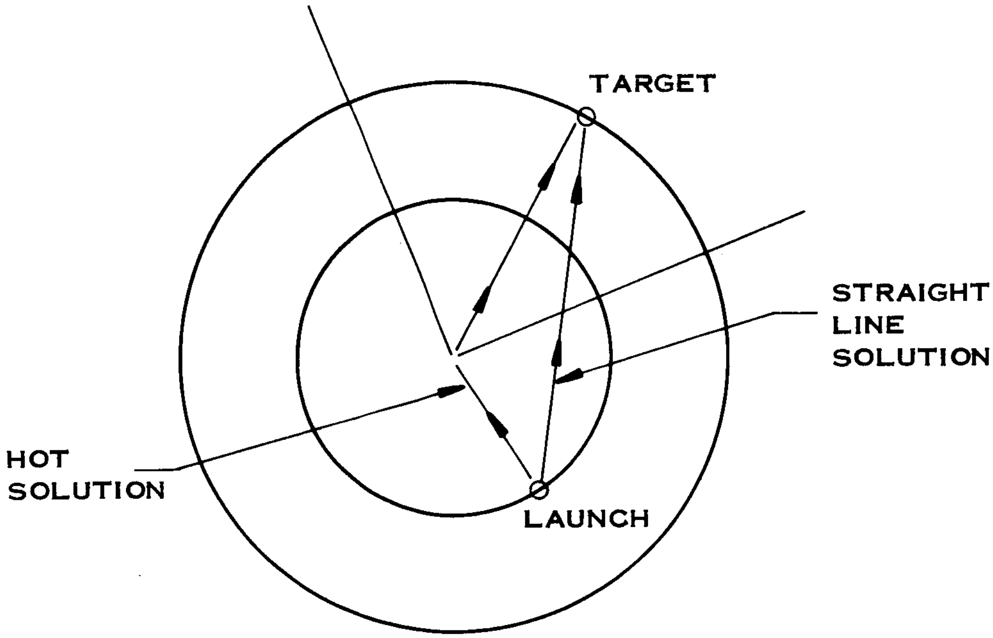
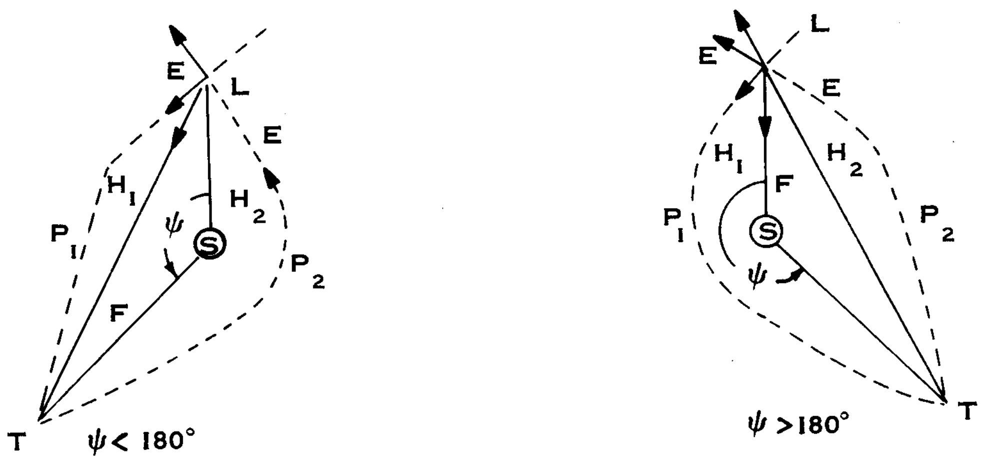


Figure A-2
Zero-Time Solutions



Heliocentric Transfers of Interest
Figure A-3

$$r_1 = \frac{p}{1 + e \cos \theta}$$

$$r_2 = \frac{p}{1 + e \cos (\theta + \psi)} \qquad \text{(A.2)}$$

The second part of Equation (A.2) may be arranged to solve for the semi-latus rectum.

$$p = r_2[1 + e \cos (\theta + \psi)]$$

$$= r_2[1 + e \cos \theta \cos \psi - e \sin \theta \sin \psi] \qquad \text{(A.3)}$$

The first of Equations (A.2) renders

$$e \cos \theta = \left(\frac{p}{r_1} - 1 \right)$$

and it is easily shown that

$$e \sin \theta = - \frac{p}{r_1} \tan \gamma$$

where γ is the flight path angle at launch.

Conic sections connecting $\vec{r}_1$ and $\vec{r}_2$ are characterized by semi-latus recta of the form

$$p = r_2[1 + \left(\frac{p}{r_1} - 1\right) \cos \psi - \frac{p}{r_1} \tan \gamma \sin \psi]$$

or

$$p = \frac{r_1 r_2 (1-\cos \psi)}{(r_1 - r_2 \cos \psi + r_2 \sin \psi \tan \gamma)} \qquad \text{(A.4)}$$

The quantities r_1, r_2 and ψ are constants for the problem. That is, p is a function of the flight path angle, γ. Only one value of γ, however, gives rise to the desired transfer time.

The transfer time between r_1 and r_2 is now computed via Kepler's Equations and compared with that desired. The difference in flight time is driven to zero through use of the iteration subroutine FINDV: The flight path angle, γ, being the parameter of iteration.

Two solutions of almost zero transfer time exist for most ephemeris configurations. The more obvious of these is the straight-line solution. The other solution is a very "hot" hyperbola which bends around the sun (see Figure A-2).

Conics between these solutions are excluded from consideration by the central force equations of motion. Trajectories which are retrograde relative to planetary motion are also excluded by limiting the range of γ allowed.

Figure (A-3) shows the trajectory regions which determine the ranges of γ to be considered, L is the launch body and T is the target body. The forbidden region, F, lines between the zero time transferred. The dotted curves P_1 and P_2 represent parabolic transfers, the regions H_1 and H_2 hyperbolic transfers, and E elliptical transfers. Retrograde transfers (H_2, P_2) are disallowed as partical solutions for this program. The initial flight path angle, γ, used to initiate the iteration is the flight path angle corresponding to the boundary solution between regions H_1 and F in the launch to target direction.

APPENDIX B

A FUNCTIONAL OPTIMIZATION PROCEDURE (FINDV)

The functional optimization procedure described in the following pages was originally written as a general purpose program and used in the Monte Carlo Program for error analysis of a lunar orbiting probe, (i.e. Contract NAS5-9700 Goddard Space Flight Center, NASA). Since then it has found many applications one of which is the flyby mission program. FINDV is used in two basic applications in this program:

1. flight time convergence criteria on the heliocentric portion of the trajectories, and
2. obtaining optimum solutions for the performance index specified.

Program input and output, and operational characteristics are described below.

PURPOSE

To find a local minimum, maximum, or zero of a function (F(X)) of a scalar (X). The subroutine itself takes only one step toward the chosen goal for each entry, and must be used with a program with provides values of F(X) and re-entries to FINDV until convergence or failure.

INPUT AND OUTPUT

I/Ø	Symbolic Name or Location	Program Dimensions	Math Symbol	Definition
I/Ø	XI	6	X	Scalar X and boundaries of interval for search.
I/Ø	FXI	6	F(X)	Function value and storage.
I/Ø	IC	6		Option Keys, number of iterations, etc.

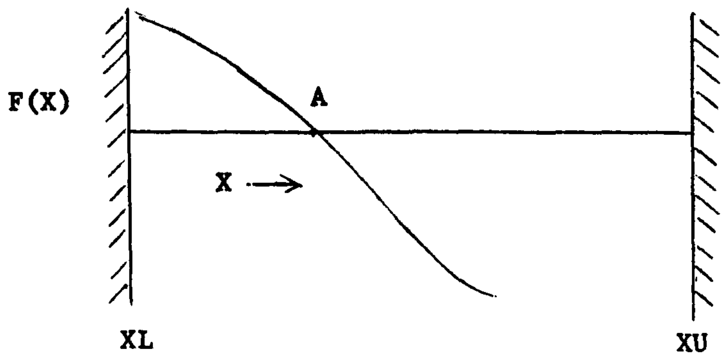
INPUT AND OUTPUT DESCRIPTION

Quantities are input or output in the XI, FXI, IC arrays as follows:

I/Ø	XI(1)	= XO	=	value of X. An initial value must be input.
I	XI(2)	= XL	=	lower boundary of X
I	XI(3)	= XU	=	upper boundary of X FINDV will search only on (XL, XU)
Ø	XI(4)	= SP	=	last value of X
Ø	XI(5)	= XPP	=	next to last value of X
Ø	XI(6)	= XØ	=	tolerance used on incremental change of X = FX(3) * .025
I	FXI(1)	= FX	=	value of F(X). Must be computed and input before each entry.
I	FXI(2)	= E	=	tolerance on solution
I/Ø	FXI(3)	= SS	=	initial step size in X, changed at each entry to size of last step taken
Ø	FXI(4)	= FP	=	F(XP) = F(XI(4))
Ø	FXI(5)	= FPP	=	F(XPP) = F(XI(5))
Ø	FXI(6)	= FØ	=	starting value of F(X) = FX(1) at first step
I/Ø	IC(1)	= IK	=	-1 on initial entry to FINDV, set by user. From then on, IK is set by FINDV: IK = 0, normal exit for new F(X) IK =-1, convergence, X and F(X) in XI(1), FXI(1). IK = 1, exceeded maximum number of iterations IK = 2, program has tried three times to step past the boundary

THEORY

Case (1) M = 0 Finding zero value of a function



In this instance we are interested in finding that value of X, X = A, such that F(X) = 0.

The program must be started with a value of X and a step which has the correct sign.

For example, if XO = XL then SS = Positive
if XO = XU then SS = Negative

In this case the subroutine will continue incrementing X by SS until either

a) the product of the starting value of F(X) and the current value of F(X) changes sign.

b) One of the boundary values of X is reached. For this limit the subroutine returns with IK = 2 if it attempts to step past the boundary twice, or finds no sign change with (XL,XU).

For nominal operation (if started correctly the condition (a) is reached.

I IC(2) = NM = maximum number of steps

Ø IC(3) = N = number of steps taken

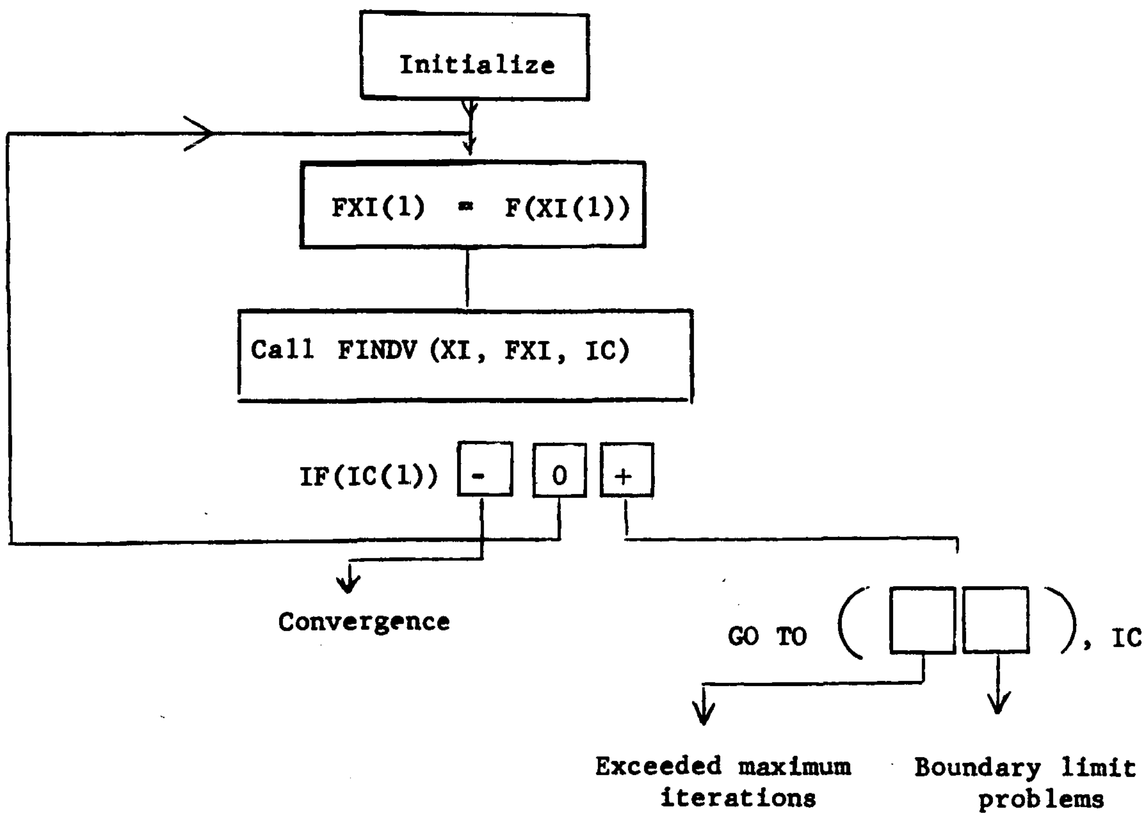
I IC(4) = M = option key. M = 0 to find a zero of F(X)
= 1 to find a minimum of F(X) or maximum of -F(X)

Ø IC(5) = JK = 1 if FINDV is in step mode
= 2 if FINDV is in parabolic mode.

Ø IC(6) = IB = 0 the fit is away from the boundary
> 0 the fit is against the boundary.

Note that to initialize FINDV, XI(1-3), FXI(2 and 3), and IC(1,2, and 4) must be set.

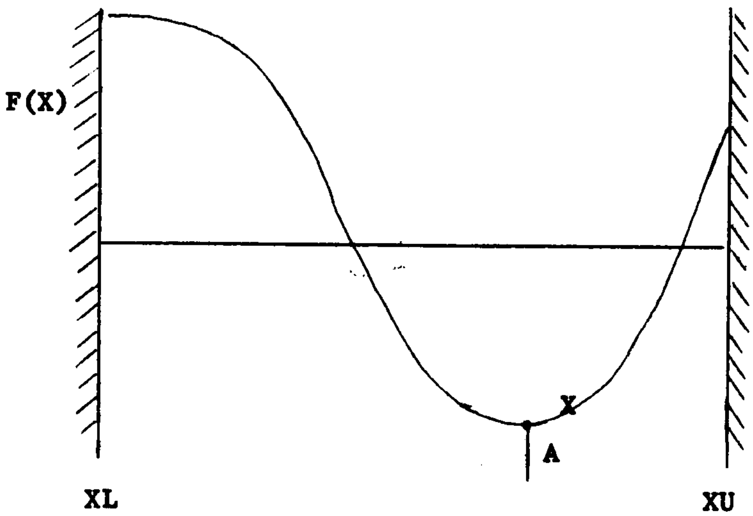
The mode of utilization in a program is illustrated below:



This last logic insures that past values of FX and X are always kept which span the desired solution.

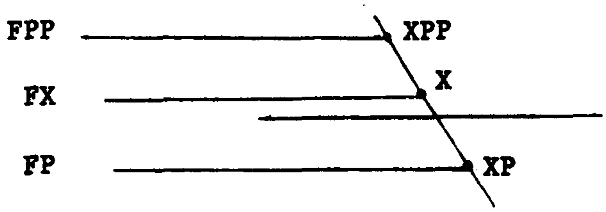
Re-entry to FINDV after computing $F(X + \delta X)$ will start with the $| FX | < \epsilon$ test.

Case (2) M = 1 - Minimizing



In this instance it is desired to find X = A, that value of X which minimizes F(X).

Following this X is incremented by a value of -SS/2 and subroutine returns



(The subroutine continually stores the last two values of FX and X and the latest value of SS. The initial SS is destroyed and not restored.)

On re-entry to FINDV, a test is made on $|FX| < \epsilon$. If the inequality holds, IK is set -1 and the subroutine returns. If not, then the three values of FX are used to find a parabola

$$F(\delta X) = a_o + a_1\ \delta X + a_2\ \delta X^2$$

where $a_o = FX$

and a_1 and a_2 are found from

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} (XP - X) & (XP - X)^2 \\ (XPP - X) & (XPP - X)^2 \end{pmatrix}^{-1} \begin{pmatrix} FP - FX \\ FPP - FX \end{pmatrix}$$

The incremental value of X from the center point is computed from

$$\delta = \frac{-a_1 + (\text{signf}\ (a_1))\ \sqrt{a_1^2 - 4a_2\ FX}}{2a_2}$$

Before making this calculation, FX, FP, and FPP are tested and, if necessary re-ordered, so that the inequality $|FX| \leq |FP| \leq |FPP|$ holds. (The X, XP and XPP are shifted correspondingly in case of re-ordering.)

The program is started similarly. At least two increments of δS are added to XX before any other tests are made.

This to give X and FX = current values
XP and FP = 1st previous values
XPP and FPP = 2nd previous values

The subroutine then contines stepping X and returning past values of X and FX until

(a) $FP - \frac{(FPP + FX)}{2} < 0$

and

(b) $(FP - FX) < 0$

or

(c) A boundary value of X is reached. In this instance two trials will be made of finding a solution close to the boundary before the subroutine returns with IK = 2.

For normal operation when conditions (a) and (b) are both reached, then XP will be near the minimum value of X.

In this instance a_1 and a_2 are computed in the manner give for case (1) and δX is computed from

$$\delta X = -a_1/2a_2, \text{ and tested against } DET = \frac{|XP - X| + |XPP - X|}{2}$$

If $|\delta X| > DET$, $|\delta X| = -\left(\text{signf}\left(\frac{FP-FX}{XP-X}\right)\right)$. DET = SS

If $|\delta X| \leq DET$, and (a) this is the first entry for which three values of F(X) have been accumulated, or (B) $|\delta X| > \frac{SS}{4.0}$ initial then SS = δX and the routine exits for a new value of F(X).

If $|\delta X| \leq DET$, and also $\leq \frac{\text{SS initial}}{4.0}$, then a test for convergence is made.

If $\left|A_1\delta X + A_2\ \delta X^2\right| \leq E + .0001\ \ F(X)$ then a solution has been reached and FINDV returns with IC(1) = -1.

Modification of step size near a boundary.

If X + δX lies outside of the open interval (XL,XU) the routine recomputes δX = SS = .9(XL-XP) or .9(XU-XP), and returns. If this occurs IB is increased by 1; three such adjustments and FINDV sets IK = 2 before exiting. (These adjustments need not have been made on consecutive steps, or at the same boundary).

APPENDIX C

OPTIMUM ΔV COMPUTATIONS ON FLYBY HYPERBOLA

C.1 ΔV COMPUTATIONS ON THE FLYBY HYPERBOLA

The equations needed for the computation of the corrective velocity vector to be added to the existing state vector in order to meet the hyperbolic departure constraints at the sphere of influence will be derived here. Figure C-1 will be used to describe the departure geometry and the symbols pertinent to the following derivation.

As Hollister and Prussing state in their paper, it is of a distinct advantage to be able to write a vector equation defining the velocity vector on the hyperbola as a function of the instantaneous radius vector and the hyperbolic departure conditions (i.e. $\vec{r}$ and $\vec{V}_\infty$ as seen in Figure C-1).

It is desired that the velocity vector be of the form

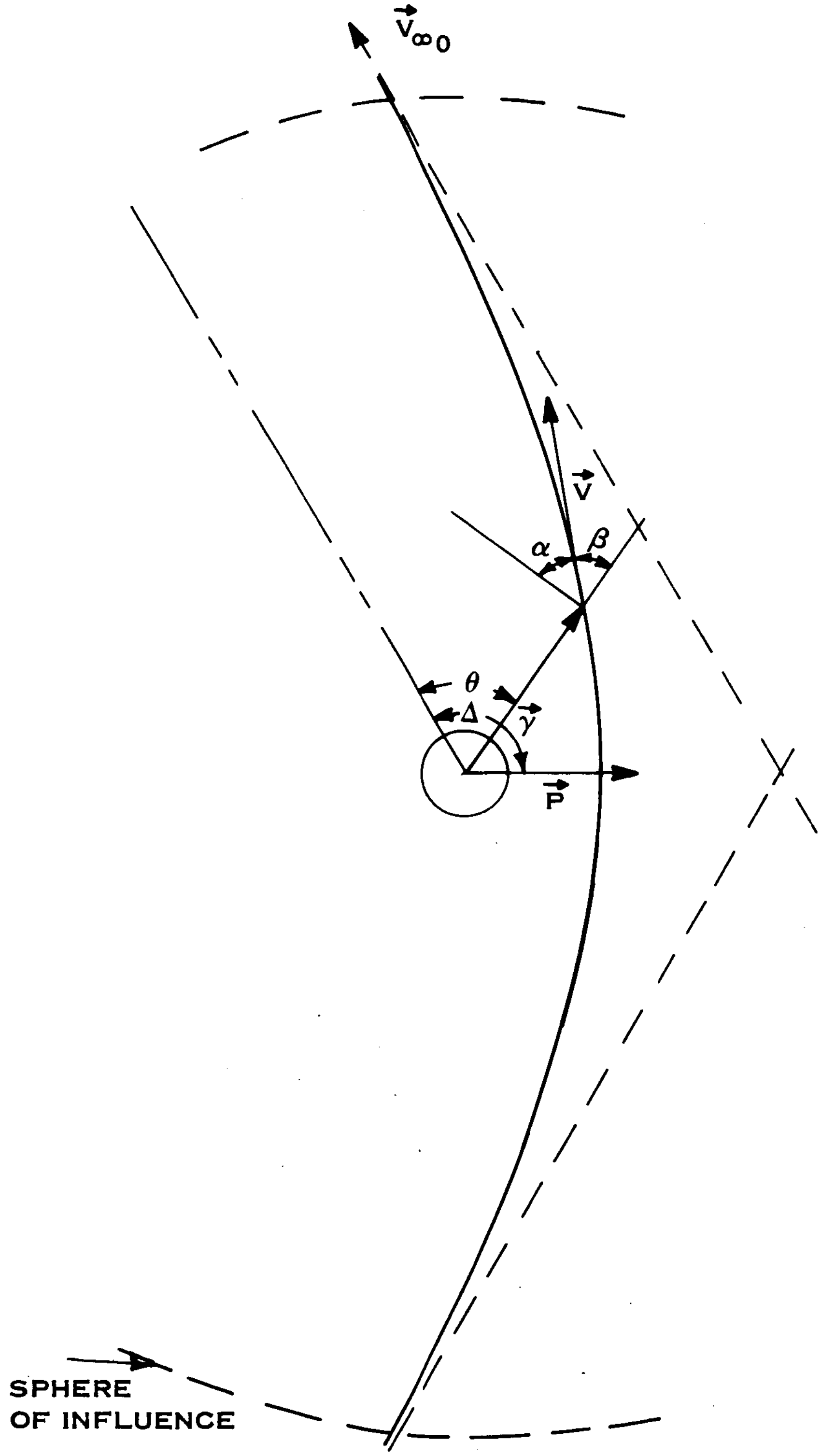
$$\vec{V} = A\vec{V}_{\infty_o} + B\,\vec{r} \tag{C.1}$$

If the $\vec{V}$ is dotted with $\vec{V}_\infty$ and $\vec{r}$, and then in turn properly manipulated, the following two equations are obtained

$$A = \frac{r^2\vec{V}\cdot\vec{V}_{\infty_o} - \vec{V}_{\infty_o}\cdot\vec{r}\;\vec{V}\cdot\vec{r}}{r^2 V_{\infty_o}^{\,2} - (\vec{r}\cdot\vec{V}_{\infty_o})^2} \tag{C.2}$$

$$B = \frac{V_{\infty_o}^{\,2}\,\vec{V}\cdot\vec{r} - \vec{V}_{\infty_o}\cdot\vec{r}\;\vec{V}_{\infty_o}\cdot\vec{V}}{r^2 V_{\infty_o}^{\,2} - (\vec{r}\cdot\vec{V}_{\infty_o})^2} \tag{C.3}$$

From Figure C-1 it is noted that



Hyperbolic Velocity on a Flyby Maneuver

Figure C.1

$$\vec{V} \cdot \vec{r} = r\,V \cos \beta \qquad (C.4)$$

$$\vec{V} \cdot \vec{V}_{\infty_o} = V\,V_{\infty_o} \cos (\theta - \beta) \qquad (C.5)$$

$$\vec{V}_{\infty_o} \cdot \vec{r} = r\,V_{\infty_o} \cos \theta \qquad (C.6)$$

A and B now reduce to:

$$A = \frac{V \sin \beta}{V_{\infty_o} \sin \theta} \qquad (C.7)$$

$$B = \frac{V (\sin \theta \cos \beta - \cos \theta \sin \beta)}{r \sin \theta} \qquad (C.8)$$

Thus: $$\vec{V} = \frac{V \sin \beta\, \vec{V}_{\infty} + V (\sin \theta \cos \beta - \cos \theta \sin \beta)\, \vec{r}}{V_{\infty_o}\, r \sin^2 \theta}$$

It is now possible, using the results of the equations of motion of a particle in an inverse square field, to eliminate β from Equation C.7. The following equation may be derived through the manipulation of the equations of motion.

$$\vec{p} = \frac{\vec{v} \times (\vec{r} \times \vec{v})}{\mu} - \frac{\vec{r}}{r} \qquad (C.10)$$

where p is a constant vector of magnitude e(the conic eccentricity,) directed toward the point of closest approach. μ is the universal gravitational constant of the flyby planet

That is; $$\vec{p}.\vec{r} = \frac{[\, V^2\, \vec{r} - \vec{V} \,.\, \vec{r}.\vec{V}]\, \vec{r} - \vec{r} \,.\, \vec{r}}{\mu r} \qquad (C.11)$$

$$= 1 + \epsilon \cos \Delta \cos \theta + \epsilon \sin \Delta \sin \theta \qquad (C.12)$$

But $$\cos \Delta = - \frac{1}{\epsilon} \rightarrow \sin \Delta = \frac{1}{\epsilon} \sqrt{\epsilon^2 - 1} \qquad (C.13)$$

therefore: $$\frac{rV^2}{\mu} \sin^2\beta = 1 - \cos\theta + \sqrt{\epsilon^2 - 1}\,\sin\theta \tag{C.14}$$

Note: $$\sqrt{\epsilon^2 - 1} = \frac{h}{\sqrt{\mu a}} = \frac{r V_{\infty_o} V \sin\beta}{\mu} \tag{C.15}$$

As a result of C.15, C.14 may be solved for sin β.

or $$\sin\beta = \frac{V_{\infty_o}}{2V} \sin\theta \left\{ 1 + \sqrt{1 + \frac{4\mu}{r V_{\infty_o}^2 (1 + \cos\theta)}} \right\} \tag{C.16}$$

Again, using Equation C.10) in the following manner yields the desired expression for (cos β sin θ - cos θ sin β)

$$\vec{p} \times \vec{r} = \frac{[\vec{v} \times (\vec{r} \times \vec{v})] \times \vec{r}}{\mu} - \frac{\vec{r} \times \vec{r}}{r} \tag{C.17}$$

$$\mu \epsilon r \sin(\Delta - \theta) = + \vec{v} \cdot \vec{r} \left| \vec{v} \times \vec{r} \right| \tag{C.18}$$

$$= r^2 v^2 \sin\beta \cos\beta \tag{C.19}$$

By substituting for sin(Δ-θ) and manpulating the results with those previously obtained, may be expressed as

$$\frac{r \sin\beta}{V} B = \frac{\dfrac{\mu \sin^2\theta}{2\, rV} + \dfrac{V_{\infty_o}}{V} \sin\beta \sin\theta \cos\theta}{\sin\theta} - \frac{\dfrac{V}{V_{\infty_o}} \sin\theta \cos\theta \sin\beta + \dfrac{\mu}{rV^2} (\cos^2\theta - \cos\theta)}{\sin\theta} \tag{C.20}$$

$$B = \frac{\mu}{rV^2}\left(\frac{1 - \cos\theta}{\sin\theta \sin\beta}\right) \tag{C.21}$$

Substitution of the expression for sin β yields the desired result for B.

$$B = \frac{V_{\infty_o}}{2r}\left\{\sqrt{1 + \frac{4\mu}{r V_{\infty_o}^2 (1 + \cos\theta)}} - 1\right\} \tag{C.22}$$

Equation C.1 now takes on the form:

$$\vec{V} = \left\{\sqrt{1 + \frac{4\mu}{r V_{\infty_o}^2 (1 + \cos\theta)}} + 1\right\} \vec{V}_\infty + \frac{V_{\infty_o}}{2r}\left\{\sqrt{1 + \frac{4\mu}{r V_{\infty_o}^2 (1 + \cos\theta)}} - 1\right\} \vec{r} \tag{C.23}$$

Note that μ could be eliminated from the expression for $\vec{V}$ by appropriately non-dimensionalizing the position and velocity.

Through a single substitution of the following definitions, the expression obtained for V may be written in terms of an incoming asymptotic velocity vector ,

$$\vec{V}_{\infty_I} \hat{=} - \vec{V}_{\infty_o} \tag{C.24}$$

$$\vec{V}_I \hat{=} - \vec{V} \tag{C.25}$$

$$\vec{V}_I = \frac{1}{2}\left\{\sqrt{1 + \frac{4\mu}{r V_{\infty_I}^2 (1 + \cos\theta)}} + 1\right\}\vec{V}_{\infty_I} - \frac{V_{\infty_I}}{2r}\left\{\sqrt{1 + \frac{4\mu}{r V_{\infty_I}^2 (1 + \cos\theta)}} - 1\right\}\vec{r} \qquad (C.26)$$

Equation (C.26) may now be used to evaluate the velocity on the incoming asymptote of the planet flyby. The incremental velocity maneuver may now be computed as:

$$\Delta\vec{V} = \vec{V} - \vec{V}_I \qquad (C.27)$$

As the expression for V is not valid for $\theta > \Delta$, velocity vectors associated with maneuvers required prior to closest approach passage on the outbound leg, or after closest approach passage on the inbound hyperbola, will require a slight alteration in the computational procedure. That is, velocity vectors prior to perigee passage would be obtained by rotating solutions for positive true anomaly backwards on the trajectory to the corresponding negative true anomaly and reversing the direction of the velocity vector. The converse, in terms of true anomaly, holds true for Equation C.26.

APPENDIX D

SENSITIVITY COEFFICIENTS

This appendix contains the derivation of the appropriate formulae for obtaining the sensitivity coefficients (i.e., sensitivity of point of application of Δv maneuver to desired alteration in departure asymptote and energy) on a flyby hyperbola. Figure D-1 defines the flyby geometry and symbols used in the subsequent derivations.

The sensitivity coefficients are generated for the nominal trajectory; that is, the desired departure conditions are assumed to be a linear deviation of the hyperbola defined by approach energy $\overline{V}_H$ and the imposed mission constraints (i.e., closest approach greater than or equal to the planet radius and the resulting heliocentric orbit being an "impact" trajectory with Earth). The resulting nominal trajectory will either be one of a "grazing" nature, which implies both energy and asymptote alteration, or a flyby at some distance from the planet, which implies energy changes only at departure from the sphere of influence. Pure flyby missions are of the latter type except that they require no alteration of the flyby hyperbola.

In order to fully assess the importance of the point of application of the thrust-augmented maneuver, the following analysis was undertaken. The two basic types of maneuvers desired at departure from the sphere of influence were used to define terminal conditions in the linear analysis. The first type of preferred maneuver was rotation of the asymptote and the second type of preferred maneuver was the alteration of departure C_3 only.

The reference coordinate system chosen for the study was the $\hat{N},\hat{V},\hat{W}$ system shown in Figure D-1: $\hat{V}$ lies along the departure asymptote, $\hat{W}$ along the orbital angular momentum vector, and $\hat{N}$ completing the orthogonal coordinate system. This system was chosen for the resulting ease of description of the desired alteration in terminal conditions.

The nominal departure velocity vector can now be written as:

$$\overline{V}_H' = V_H \hat{V} \qquad \text{D.1}$$

The incoming velocity vector is defined by a simple rotation of magnitude $-\psi$ (see Figure D-1) about the orbital angular momentum vector $\hat{W}$.

$$\overline{V}_H = T_{\hat{W}} (-\psi) \overline{V}_H' \qquad \text{D.2}$$

where $T_{\hat{W}} (-\psi)$ is mathematical rotation for the rotation described and is of the form

$$T_{\hat{W}} (-\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{D.3}$$

The complete state at patch may be defined through the use of D.3 and the following expression for the radius vector:

$$\overline{R} = T_{\hat{W}} (\beta) \overline{R}_{Patch} , \overline{R}_{Patch} = r_{Patch} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{D.4}$$

where r_{Patch} = Patch radius

$$\beta = \sigma + \psi - \Phi_{Patch}$$

and $\Phi_{Patch} = \cos^{-1} \frac{1}{e} \left[\frac{p}{r_{Patch}} - 1 \right]$

A rotation of $\overline{R}$ through the angle 2σ will provide the position state on entry into the sphere of influence.

$$\bar{R}' = T_{\hat{W}}(2\sigma)\,\bar{R} \qquad \text{D.5}$$

The state at entry into the sphere of influence is now completely defined and serves as the starting point for the sensitivity analysis. The independent parameter in the analysis will be the true anomaly, φ, rather than time. The sensitivity coefficients will be generated as a function of true anomaly where initial condition is defined to be $-\varphi_{Patch}$. As a result, the state at some intermediate point may be computed as follows. It is assumed that the present state is arrived at through a sequential stepping of Φ from $-\Phi_{Patch}$ to $(-\varphi_{Patch} + N\,\Delta\Phi)$, where N is the number of $\Delta\Phi$ increments the true anomaly has been advanced. Each sequential stepping of $\Delta\Phi$ will define the state at the terminus of that $\Delta\Phi$ increment in terms of the known state at the beginning of that $\Delta\Phi$ increment. That is, the state after N $\Delta\varphi$ steppings can be written as:

$$\bar{R}_{N+1} = f(\varphi_N + \Delta\varphi)\,\bar{R}_N + g(\varphi_{N-1} + \Delta\varphi)\,\bar{V}_N \qquad \text{D.6}$$

$$\bar{V}_{N+1} = \dot{f}(\varphi_N + \Delta\varphi)\,\bar{R}_N + \dot{g}(\varphi_{N-1} + \Delta\varphi)\,\bar{V}_N \qquad \text{D.7}$$

where $\bar{R}_N \Big|_{N=0} = \bar{R}$ and $\bar{V}_N \Big|_{N=0} = \bar{V}$; the entry conditions onto the sphere of influence.

The coefficients in D.6 and D.7 are scalar quantities defined in terms of the initial conditions (i.e., $\bar{V},\bar{R}$) and the current value of Φ. The detailed derivations of these expressions may be obtained from many sources, one of which is Reference 6.

As the sensitivity coefficients and the transition matrix are directly related, it becomes a necessity to know the time interval over which the transition matrix must be computed. Inherent to the method of computation of the transition matrix is the requirement of elapsed time. In the existing version of the program, time between the current state and the departure state is computed via Lambert's Equation.

The deviation state at time t_{N+1} may now be related to the deviation state at time t_F as

$$\begin{pmatrix} \delta\bar{R}' \\ \delta V' \end{pmatrix} = \Phi \left(t_F, t_{N+1} \right) \begin{pmatrix} \delta\bar{R}_{N+1} \\ \delta V_{N+1} \end{pmatrix} \qquad \text{D.8}$$

As it is wished to relate deviation state at time t_{N+1} to a particular deviation at time t_F, equation D.8 must be rewritten as:

$$\begin{pmatrix} \delta\bar{R}_{N+1} \\ \delta\bar{V}_{N+1} \end{pmatrix} = \Phi^{-1} \left(t_F, t_{N+1} \right) \begin{pmatrix} \delta\bar{R}' \\ \delta\bar{V}' \end{pmatrix} \qquad \text{D.9}$$

Furthermore, it is of a distinct advantage to look at the sensitivity coefficients evaluated in a local $\hat{N},\hat{V},\hat{W}$ system. This system is termed the $\hat{n},\hat{v},\hat{w}$ system and is defined in Figure D-1. Resolving the state into $\hat{n},\hat{v},\hat{w}$ coordinates through the transformation $T_{n,v,w}$ yields

$$\begin{pmatrix} \delta\bar{r}_{N+1} \\ \delta\bar{v}_{N+1} \end{pmatrix} = T_{n,v,w} \begin{pmatrix} \delta\bar{R}_{N+1} \\ \delta\bar{V}_{N+1} \end{pmatrix} \qquad \text{D.10}$$

Equation D.9 may now be written as

$$\begin{pmatrix} \delta\bar{r}_{N+1} \\ \delta\bar{v}_{N+1} \end{pmatrix} = T_{n,v,w} \, \Phi^{-1} \left(t_F, t_{N+1} \right) \begin{pmatrix} \delta\bar{R}' \\ \delta\bar{V}' \end{pmatrix} \qquad \text{D.11}$$

The foward transition matrix results from integration of the variational equations for an inverse square force field resolved in an orthogonal coordinate system. For these conditions, a closed form set of solutions exists. However, the inverse transition matrix for such a system can be shown to be

$$\Phi^{-1}\left(t_F, t_{N+1}\right) = \begin{bmatrix} \Phi_4^{\ T} & -\Phi_2^{\ T} \\ -\Phi_3^{\ T} & \Phi_1^{\ T} \end{bmatrix} \qquad \text{D.12}$$

where $$\Phi\left(t_F, t_{N+1}\right) = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix} \qquad \text{D.13}$$

By defining the matrix $T_{N,V,W}\ \Phi^{-1}\left(t_F, t_{N+1}\right)$ as α, we have the following:

$$\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \hat{=} \ T_{N,V,W}\ \Phi^{-1}\left(t_F, t_{N+1}\right) \qquad \text{D.14}$$

Using D.14 and the constraint equation that

$$\delta\bar{r}_{N+1} = 0 \qquad \text{D.15}$$

the expression

$$0 = \alpha_1\ \delta\bar{R}' + \alpha_4\ \delta\bar{V}' \qquad \text{D.16}$$

or $$\delta\bar{R}' = -\alpha_1^{-1}\alpha_2\ \delta\bar{V}'$$

is obtained.

Substituting expression D.16 into the expression for $\delta\bar{v}_{N+1}$ yields:

$$\delta\bar{v}_{N+1} = \alpha_3\ \delta\bar{R}' + \alpha_4\ \delta\bar{V}' \qquad \text{D.17}$$

$$= \alpha_3\ -\alpha_1^{-1}\alpha_2\ \ \delta\bar{V}' + \alpha_4\ \delta\bar{V}'$$

$$= \left[\alpha_4 - \alpha_3\alpha_1^{-1}\alpha_2\right]\delta\bar{V}' \qquad \text{D.18}$$

$$\hat{=}\ \beta\ \delta\bar{V}'$$

where

$$\beta = \begin{bmatrix} \beta_1 & \beta_2 & \beta_5 \\ \beta_3 & \beta_4 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \end{bmatrix} \qquad \text{D.19}$$

However, only the inplane components of the sensitivity matrix β are of direct interest (i.e., β_1, β_2, β_3, β_4).

The on-orbit sensitivity to energy changes at the sphere of influence may now be investigated by defining a unit error in the $\hat{V}$ direction. In essence,

$$\left(\frac{\delta\bar{v}_{n+1}}{\delta\bar{V}_{V}{}'}\right) = \begin{pmatrix} \beta_2 \\ \beta_4 \end{pmatrix}, \text{ where } \delta\bar{V}' = \delta\bar{V}_{\hat{N}} + \delta\bar{V}_{\hat{V}}{}' + \delta\bar{V}_{\hat{W}}{}' \qquad \text{D.20}$$

B_4 represents the sensitivity of tangential velocity at t_{N+1} to desired changes in tangential velocity at exit from the sphere of influence.

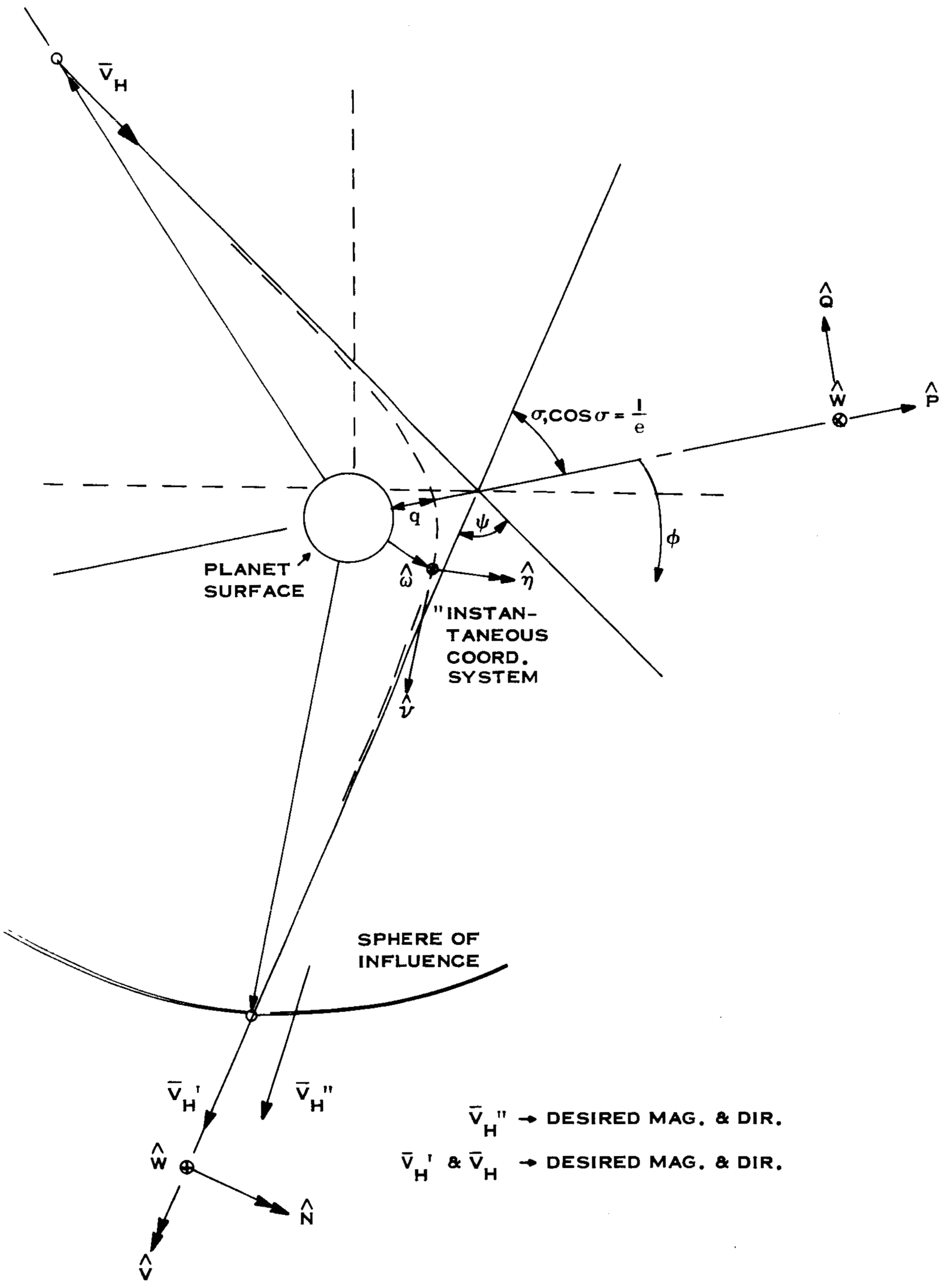
Likewise, β_2 represents the sensitivity of changes in the velocity tangential to $\hat{n}$ to desired changes in tangential velocity at exit from the sphere of influence.

In a similar manner, the sensitivity coefficients relating tangential and normal velcoity changes to desired changes in the direction of the outgoing asymptote may be written:

$$\left(\frac{\delta\bar{v}_{n+1}}{\delta\bar{V}_{\hat{N}}{}'}\right) = \begin{pmatrix} \beta_1 \\ \beta_3 \end{pmatrix} \qquad \text{D.21}$$

Checkout of the program necessary for computation of the required data is proceeding. As stated earlier in the report, it is assuming a secondary role in the overall study. However, the basic reasoning behind this investigation is to determine the magnification factor by which the ΔV

maneuver at Venus is in error when it is computed as in the present version of the swing-by mission analysis program (i.e., at the sphere of influence on the departure leg of the flyby hyperbola).



Geometry for Sensitivity Coefficicent Analysis
Figure D-1

APPENDIX E

ΔV COMPUTATIONS IN EXISTING PROGRAM

E.1 FLYBY COMPUTATIONS AT THE INTERMEDIATE BODY

The techniques for rapidly simulating the gravity-assist missions have been implemented in a digital computer program.

Having assigned the launch conditions as to latitude, longitude, azimuth and latitude of the parking orbit at the launch body, each choice of a launch date, L, and time of flight, T_1, determines the conic trajectory between the first and second body ("first leg"), and yields the hyperbolic approach asymptote $\vec{S}_1$ relative to the second body. The arrival date, $(L + T_1)$, at the second body becomes the departure date for the conic trajectory to the third or target body, ("scond leg"). The choice of a time of flight, T_2, for the second leg will determine the necessary hyperbolic departure asymptote, $\vec{S}_2$, required to satisfy that time of flight to the third body.

The transition, from the hyperbolic path (relative to the intermediate body) having asymptotes $\vec{S}_1$ and $\vec{S}_1'$, to the path with asymptote $\vec{S}_2$, will in general require a corrective velocity change $\Delta\vec{V}$ at some time during the interval of passing the intermediate body. The 3-body computer program makes the assumption that this correction is to be made as the probe leaves the sphere of influence of the intermediate body.

For present purposes, there are no constraints to satisfy as to the manner in which the probe passes the intermediate body, except for the radius of closest approach, R_p. Hence, the vectors $\vec{S}_1$ and $\vec{S}_2$ may be assumed to define the plane of the orbit relative to the intermediate body. The velocity change $\Delta\vec{V}$ will then have no out-of-plane component.

For any hyperbolic trajectory, the radius of closest approach, R_p, is related to the half-angle, β, between the two asymptotes by

$$R_p = \frac{\mu}{C_3}\left(\frac{1}{\cos\beta} - 1\right) \tag{E.1}$$

where

$$C_3 = V^2 - \frac{2\mu}{r} = \text{(twice) energy} \tag{E.2}$$

$$\mu = \text{central gravitational constat}$$

and V and r are the velocity and radius vector magnitudes at any point along the conic.

Let β_o be the half-angle between $-\vec{S}_1$ and $\vec{S}_2$. The corresponding R_p is then

$$R_o = R_p(\beta_o) = \frac{\mu}{\vec{S}_1 \cdot \vec{S}_1}\left(\frac{1}{\cos\beta_o} - 1\right) \tag{E.3}$$

Suppose R_{min} is a pre-assigned minimum approach radius for the intermediate body. If $R_o \geq R_{min}$, no change in direction will be needed as the probe leaves the sphere of influence, and $\Delta\vec{V}$ will be a change in velocity magnitude only. Let r_s be the radius of the sphere of influence of the intermediate body. Then, using Equation (E.2), at the sphere of influence

$$V_2 = \sqrt{\vec{S}_2 \cdot \vec{S}_2 + \frac{2\mu}{r_s}} = \text{required magnitude}$$

$$V_1 = \sqrt{\vec{S}_1 \cdot \vec{S}_1 + \frac{2\mu}{r_s}} = \text{uncorrected magnitude}$$

$$\left|\Delta\vec{V}\right| = \left|V_2 - V_1\right| .$$

If R_o, computed by Equation (E.3), is too small, i.e.,

$$R_o < R_{min},$$

then a direction change will be necessary as the probe leaves the sphere of influence. The probe is allowed to pass at a radius $R_p = R_{min}$. The half-angle between $-\vec{S}_1$ and the uncorrected departure asymptote is, from Equation (E.1),

$$\beta_{min} = \cos^{-1}\left(\frac{1}{\frac{C_3}{\mu} R_{min} + 1} \right)$$

The direction must ben changed by the amount $2\Delta\beta$, where

$$2\Delta\beta = 2\beta_{min} - 2\beta_o$$

At the sphere of influence, (Figure E-1), the magnitude of the velocity change is then

$$\left| \Delta\vec{V} \right| = \sqrt{(V_2 \sin 2\Delta\beta)^2 + (V_2 \cos 2\,\Delta\beta - V_1)^2}$$

In the 3-body program, in its main mode of application, $\left|\Delta\vec{V}\right|$ is considered to be a "penalty function", which is to be minimized by using the second-leg flight time T_2 as a control parameter. Variation of T_2 results in a spatial variation in the departure asymptote $\vec{S}_2$, which in turn has a direct effect on the required $\left|\Delta\vec{V}\right|$. An iterative procedure (subroutine FINDV) is employed to find the value T_2^*, with $T_{min} \leq T_2^* \leq T_{max}$, for which $\left|\Delta\vec{V}\right|$ is a minimum. The limits T_{min} and T_{max} are program input quantities.

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* T_2^* is that value of T_2 which minimizes $\Delta\vec{V}$.

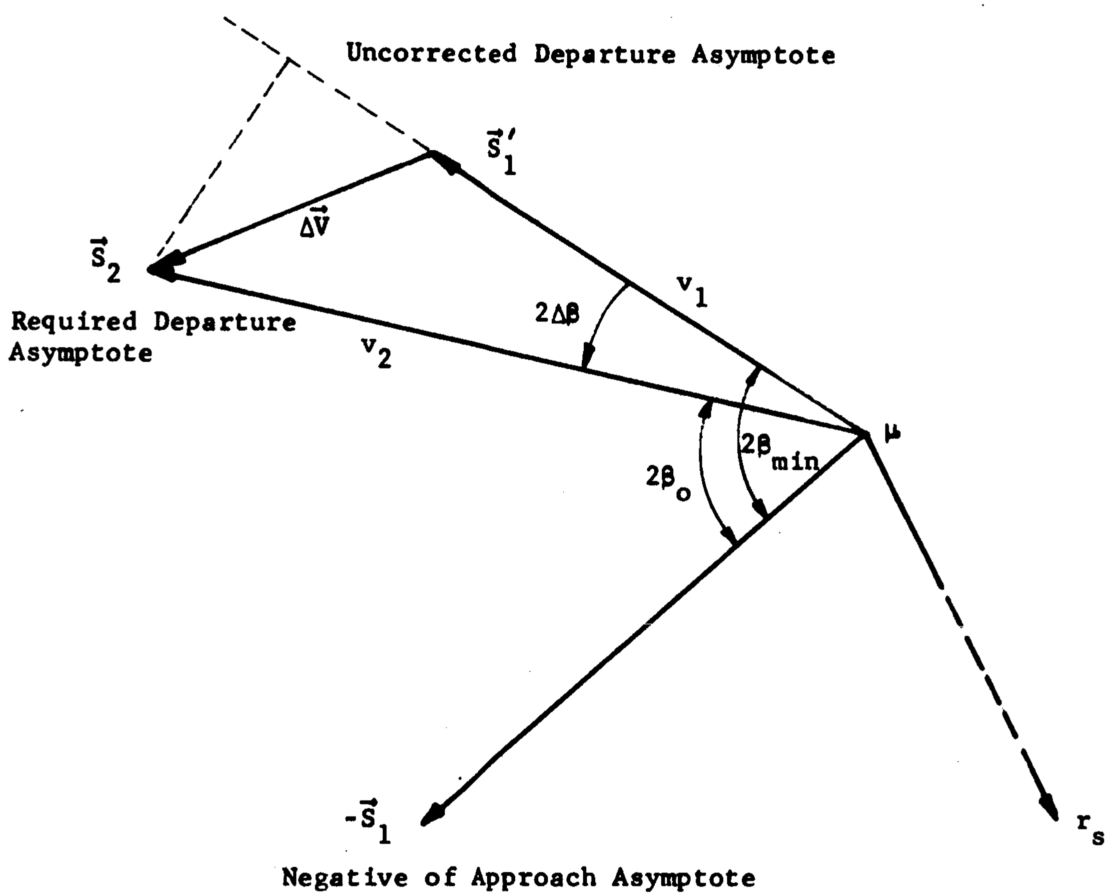


Figure E-1 Passage Geometry at Intermediate Body

APPENDIX F

PARAMETRIC DATA ON 1978 OPPORTUNITY
MARS LAUNCH DATE: 2 March 1978

The following plots represent data generated on the 2 March 1978 Mars launch of an interplanetary swing-by Venus mission for the return to Earth. The plots are essentially given on two bases: the first set of data utilizes flight time on the first leg of the mission as the parameter for displaying the information, and the second set of data utilizes the total Mars-Earth transit time as the parameter for information display. Each set contains pertinent mission parameters plotted as a function of either Mars-Venus transit time or Mars-Earth transit time (depending upon the choice of parametric display variables) and asymptotic approach velocity to Earth.

The contents of each figure are self-explanatory.

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

PLOTTING SYMBOL DEFINITIONS

PARAMETER FOR CURVES

FLIGHT TIME (MARS TO EARTH) (DAYS)

SYMBOL	PARAMETER VALUE
O	$2.4000 \times 10^{+02}$
X	$2.4200 \times 10^{+02}$
□	$2.4400 \times 10^{+02}$
Y	$2.4600 \times 10^{+02}$
+	$2.4800 \times 10^{+02}$
‡	$2.5000 \times 10^{+02}$
L	$2.5200 \times 10^{+02}$
U	$2.5400 \times 10^{+02}$
D	$2.5600 \times 10^{+02}$
H	$2.5800 \times 10^{+02}$
C	$2.6000 \times 10^{+02}$
V	$2.6200 \times 10^{+02}$
J	$2.6400 \times 10^{+02}$
Z	$2.6600 \times 10^{+02}$
●	$2.6800 \times 10^{+02}$

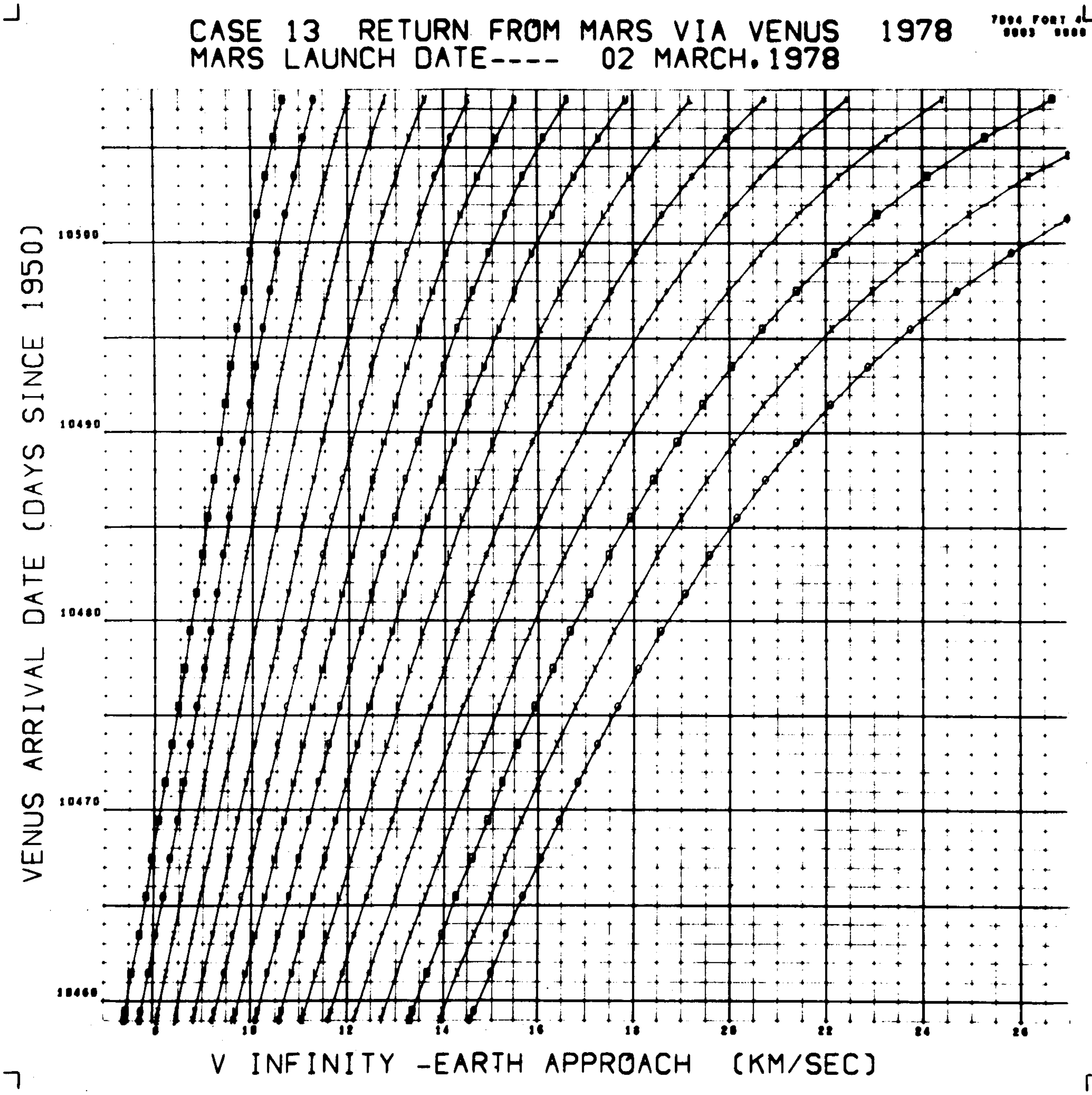
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

PLOTTING SYMBOL DEFINITIONS

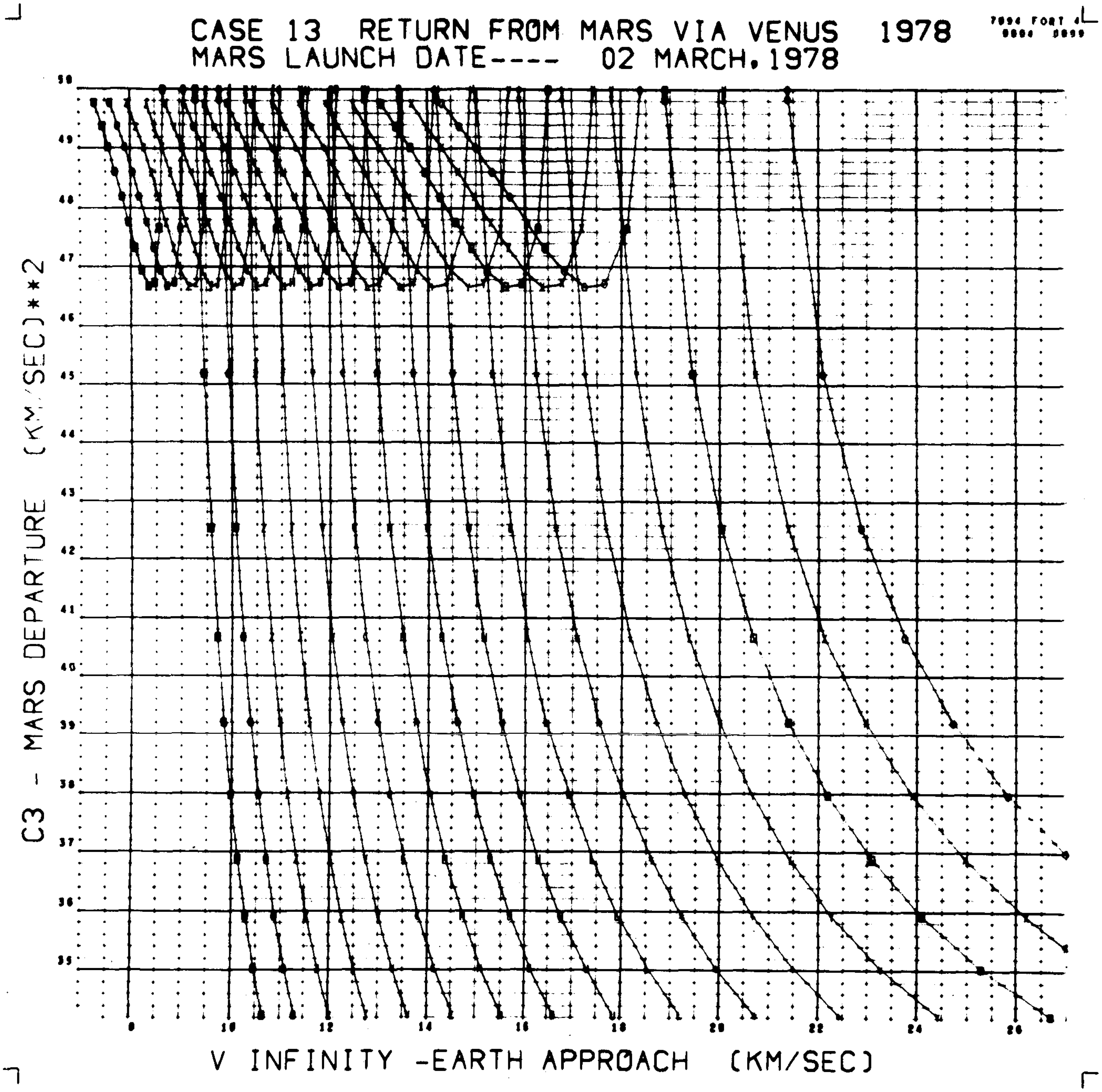
PARAMETER FOR CURVES

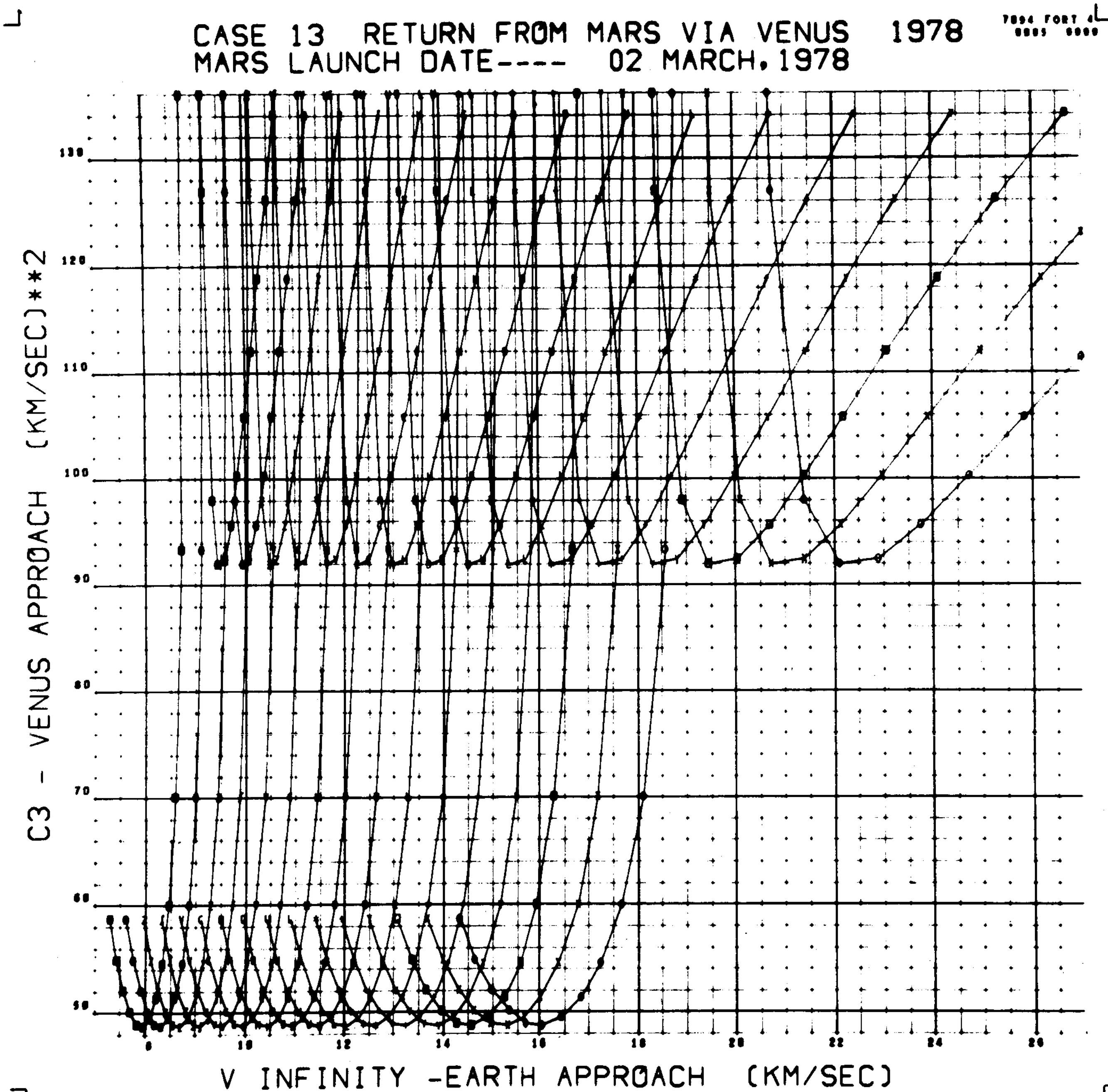
FLIGHT TIME (MARS TO EARTH) (DAYS)

SYMBOL	PARAMETER VALUE
■	$2.7000 \times 10^{+02}$

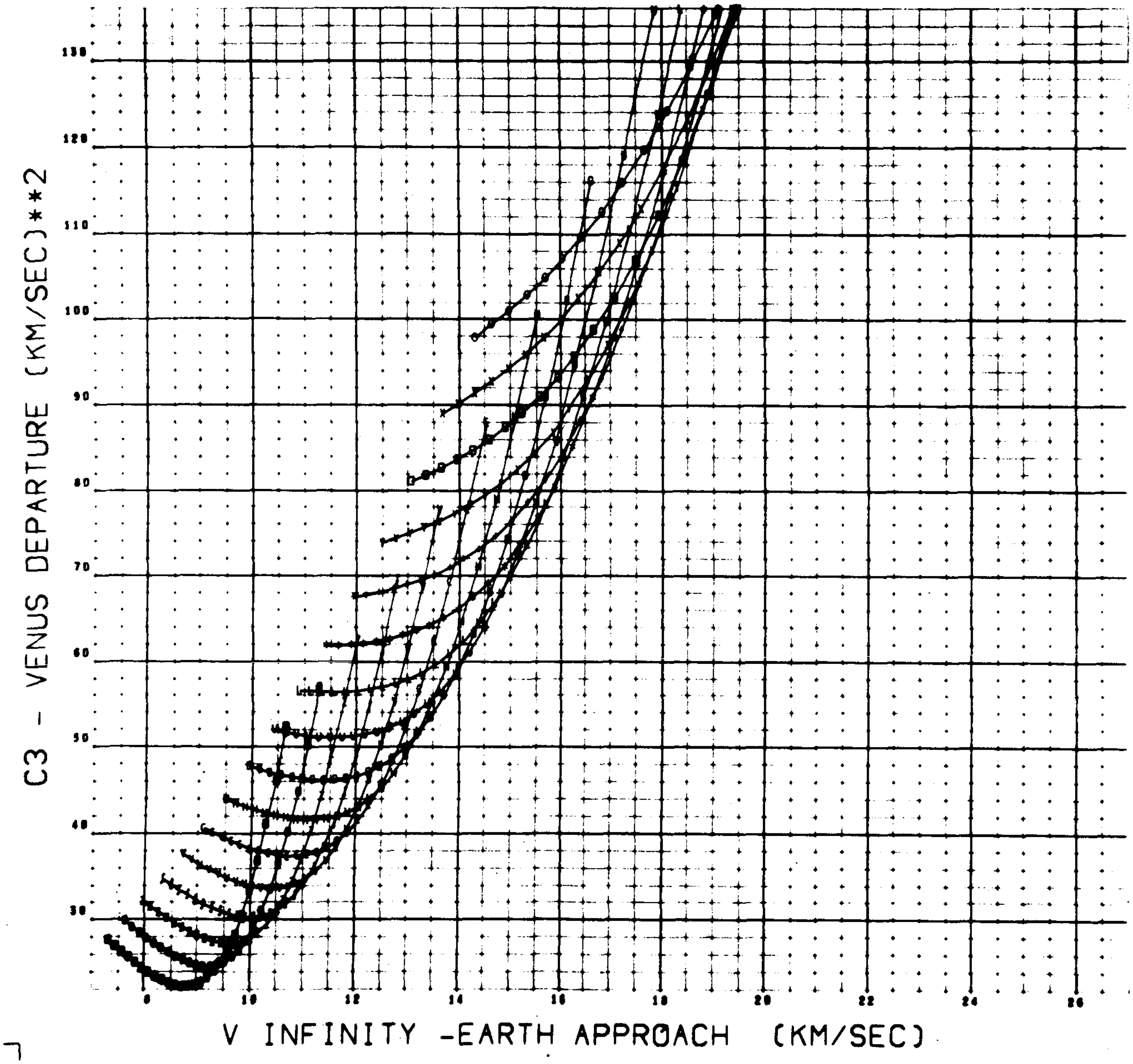


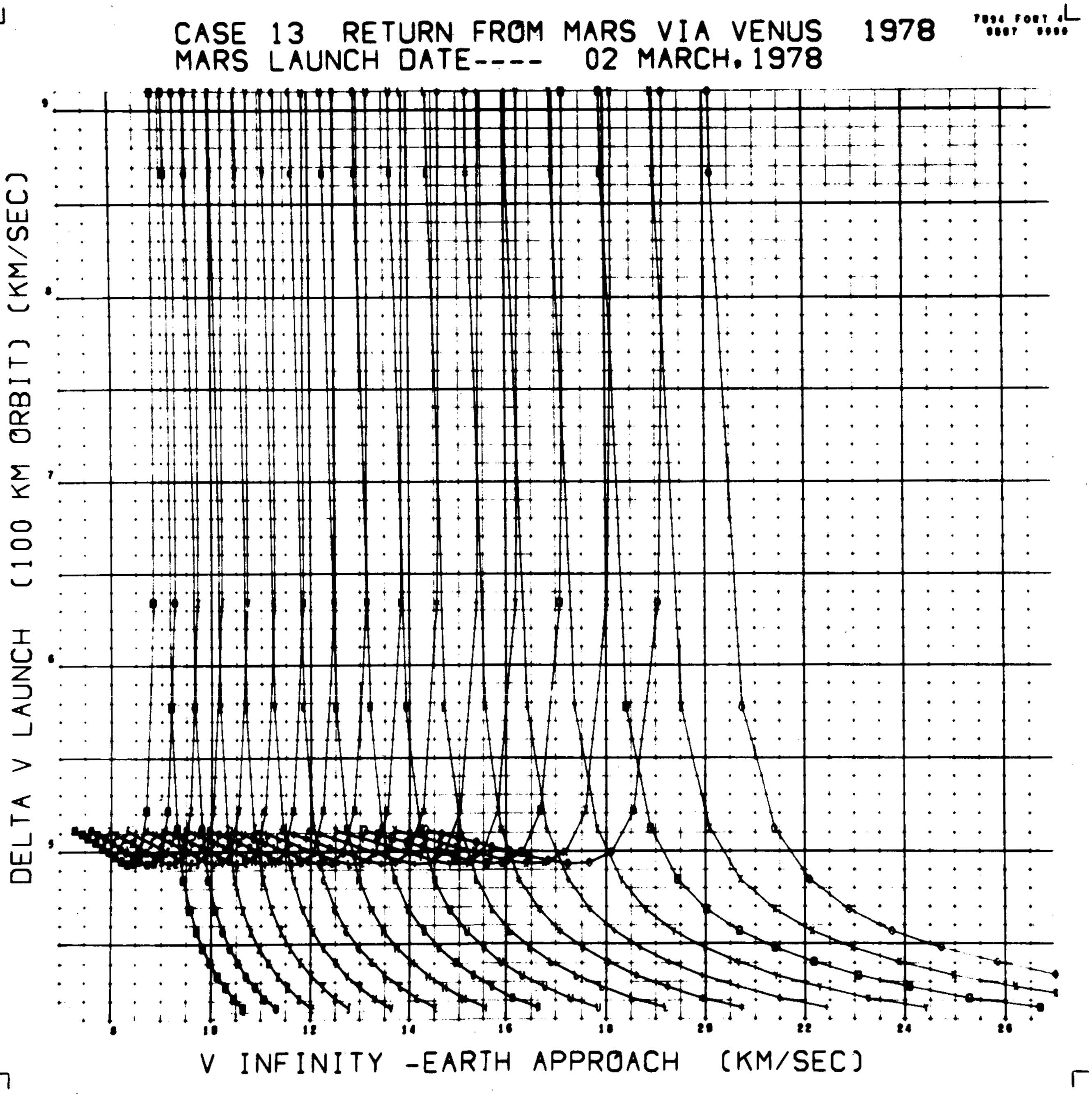
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
VENUS ARRIVAL DATE (DAYS SINCE 1950)
10500
10490
10480
10470
10460
V INFINITY -EARTH APPROACH (KM/SEC)
8
10
12
14
16
18
20
22
24
26


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
C3 - MARS DEPARTURE (KM/SEC)**2
V INFINITY -EARTH APPROACH (KM/SEC)
50
49
48
47
46
45
44
43
42
41
40
39
38
37
36
35
8
10
12
14
16
18
20
22
24
26

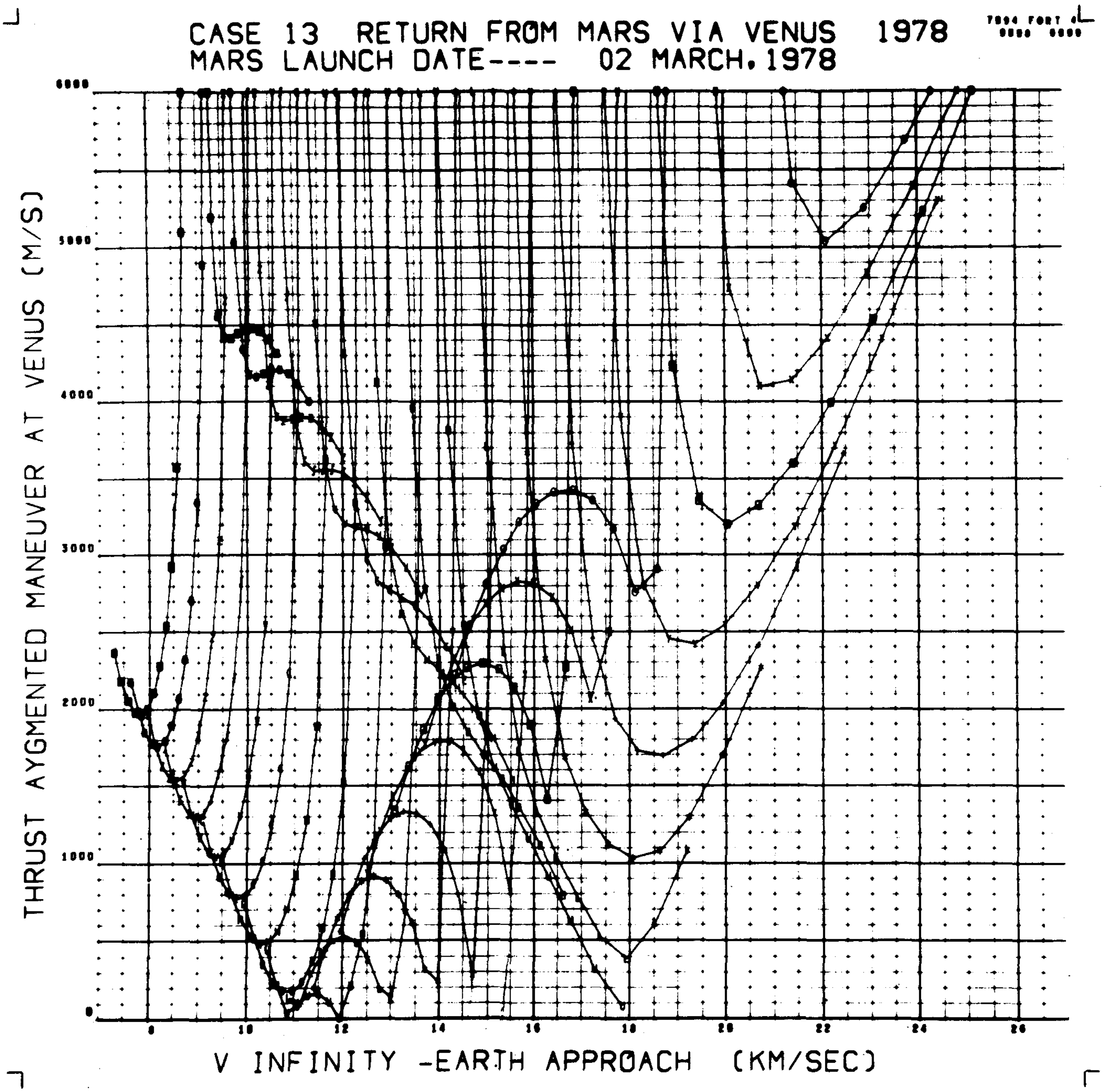

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
C3 - VENUS APPROACH (KM/SEC)**2
V INFINITY -EARTH APPROACH (KM/SEC)

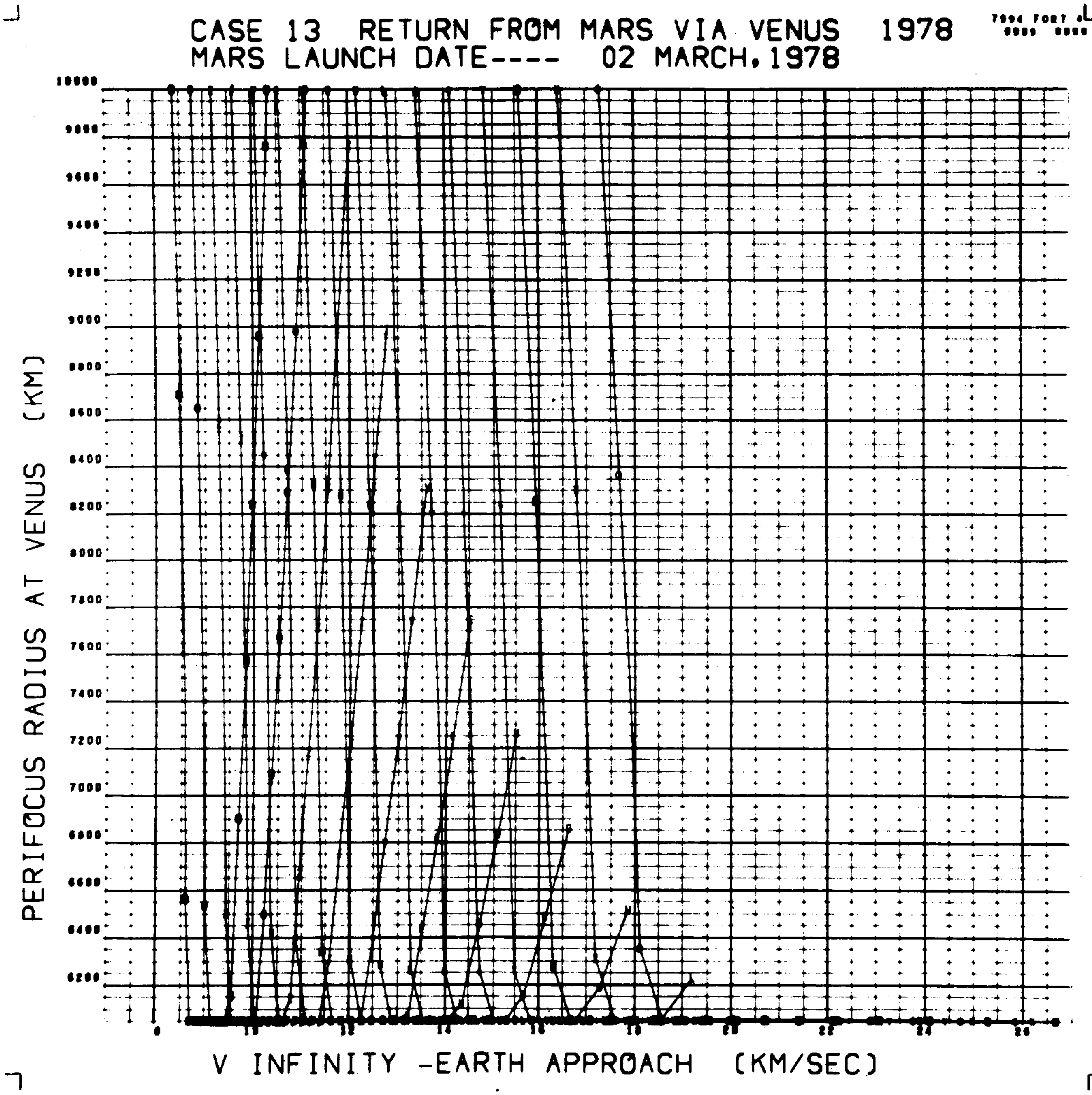
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

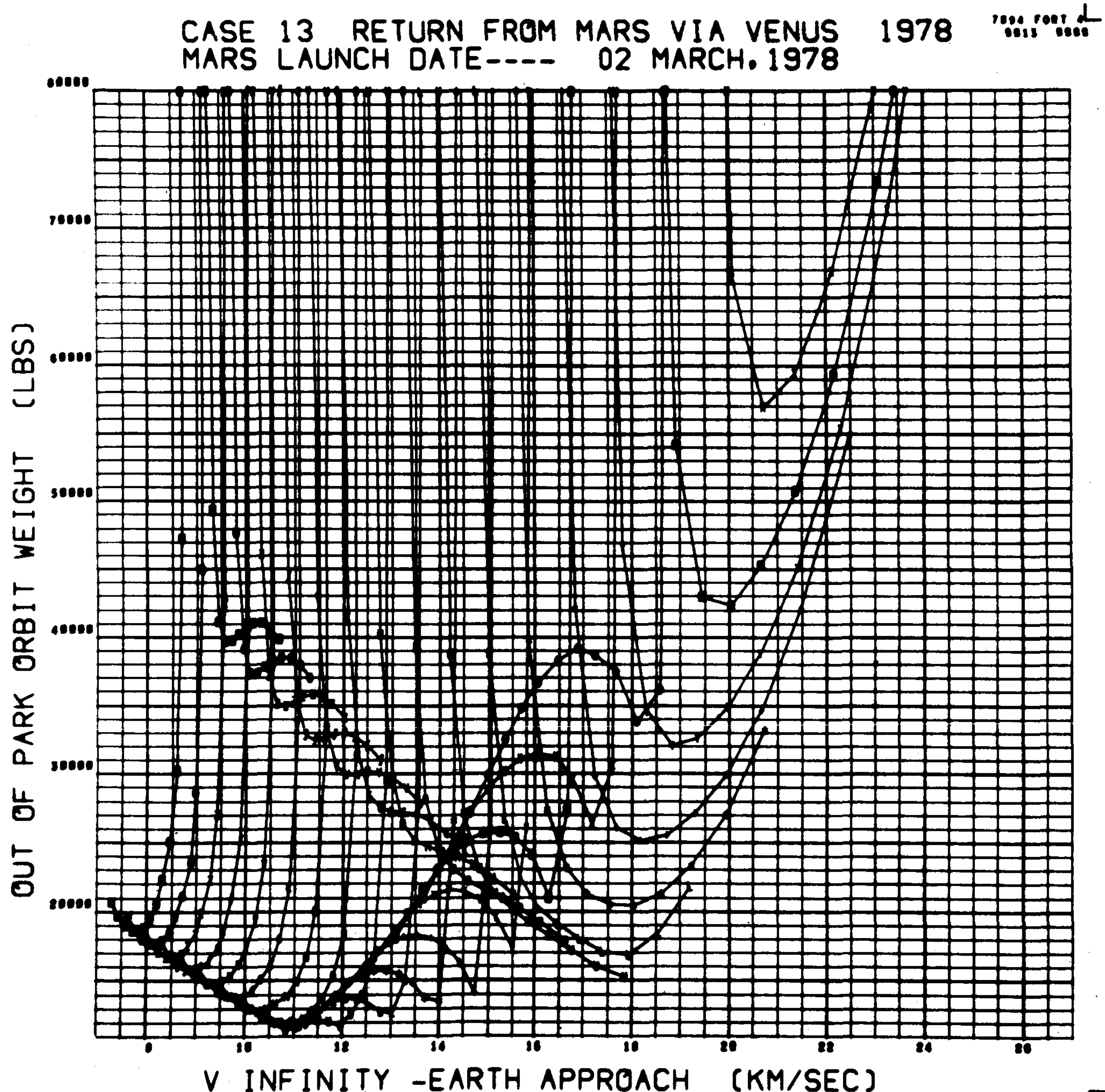


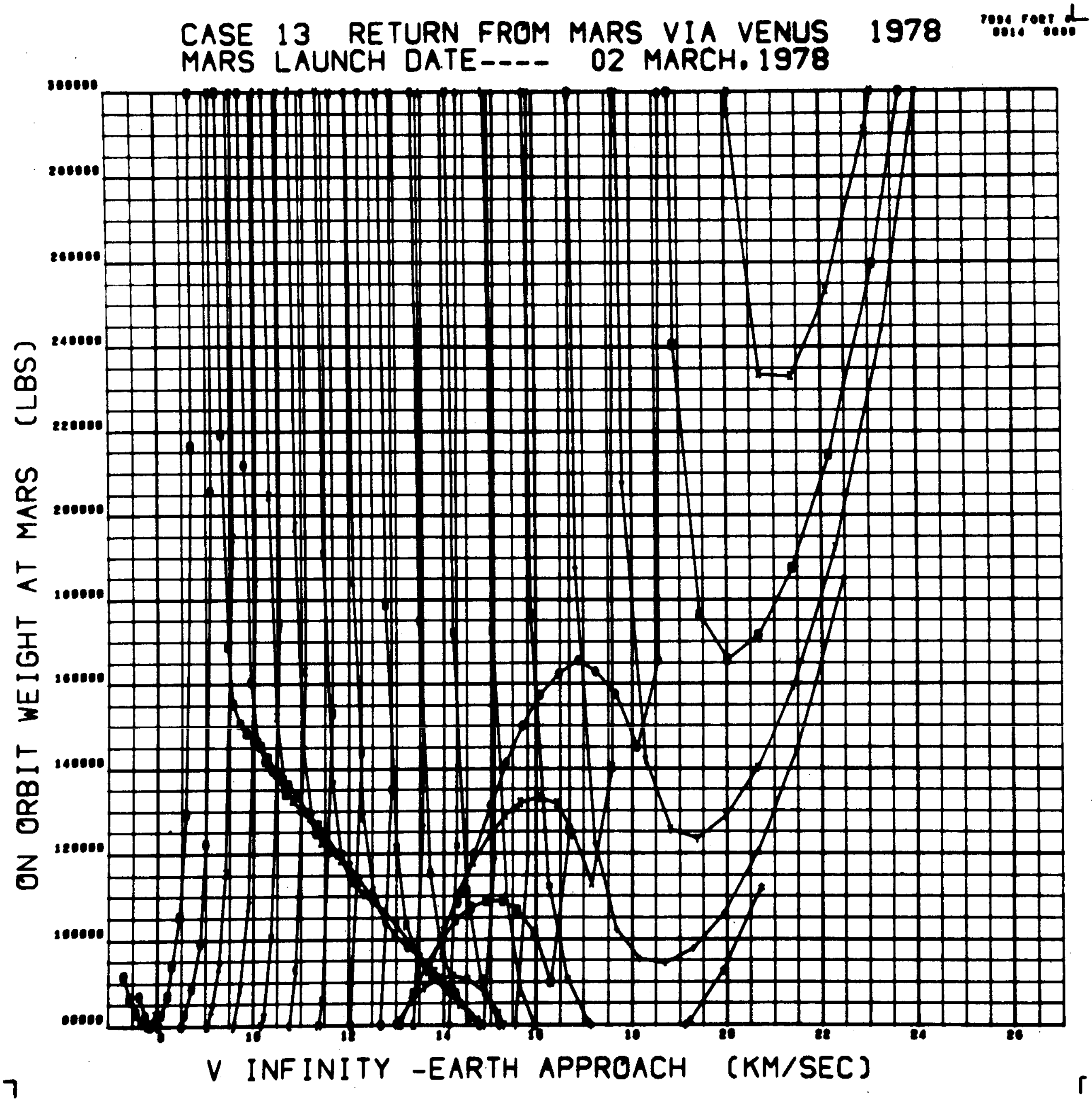


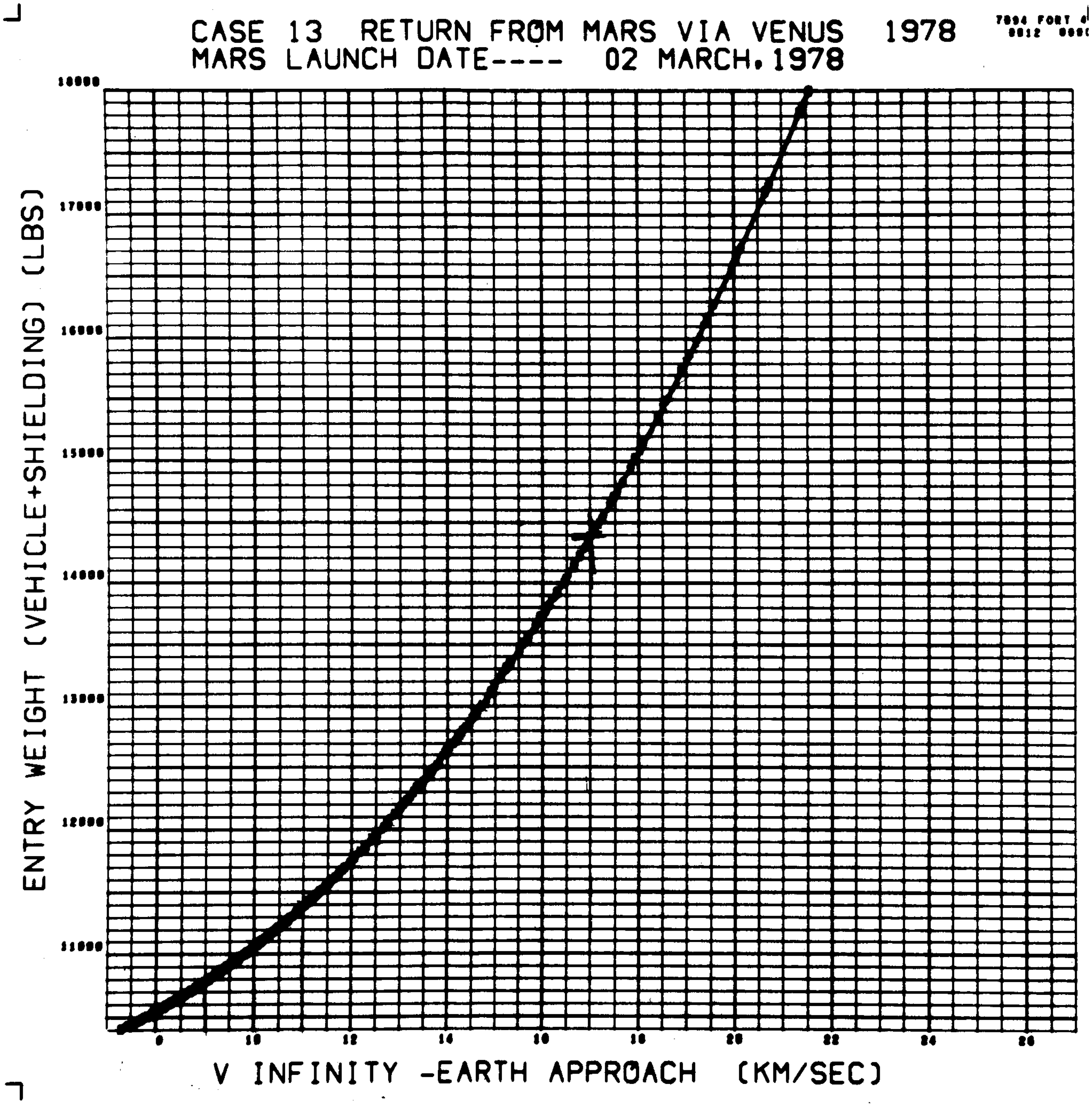
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
DELTA V LAUNCH (100 KM ORBIT) (KM/SEC)
V INFINITY -EARTH APPROACH (KM/SEC)

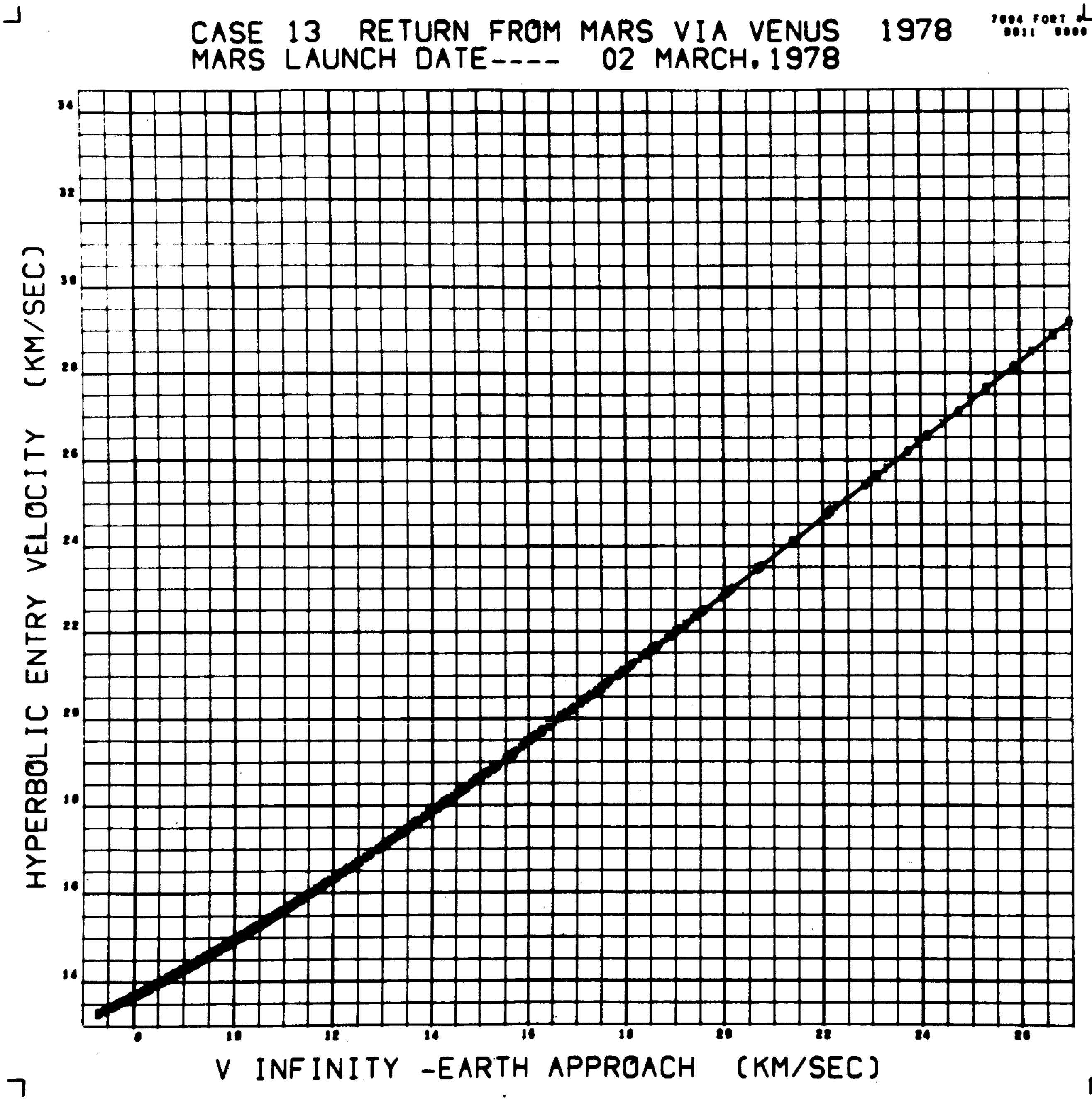

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
THRUST AYGMENTED MANEUVER AT VENUS (M/S)
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
PERIFOCUS RADIUS AT VENUS (KM)
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
OUT OF PARK ORBIT WEIGHT (LBS)
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
ON ORBIT WEIGHT AT MARS (LBS)
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
ENTRY WEIGHT (VEHICLE+SHIELDING) (LBS)
18000
17000
16000
15000
14000
13000
12000
11000
8
10
12
14
16
18
20
22
24
26
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
HYPERBOLIC ENTRY VELOCITY (KM/SEC)
V INFINITY -EARTH APPROACH (KM/SEC)

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

PLOTTING SYMBOL DEFINITIONS

PARAMETER FOR CURVES

FLIGHT TIME (MARS TO EARTH) (DAYS)

SYMBOL	PARAMETER VALUE
O	$2.4000 \times 10^{+02}$
X	$2.4200 \times 10^{+02}$
□	$2.4400 \times 10^{+02}$
Y	$2.4600 \times 10^{+02}$
+	$2.4800 \times 10^{+02}$
*	$2.5000 \times 10^{+02}$
L	$2.5200 \times 10^{+02}$
U	$2.5400 \times 10^{+02}$
D	$2.5600 \times 10^{+02}$
H	$2.5800 \times 10^{+02}$
C	$2.6000 \times 10^{+02}$
V	$2.6200 \times 10^{+02}$
J	$2.6400 \times 10^{+02}$
Z	$2.6600 \times 10^{+02}$
●	$2.6800 \times 10^{+02}$

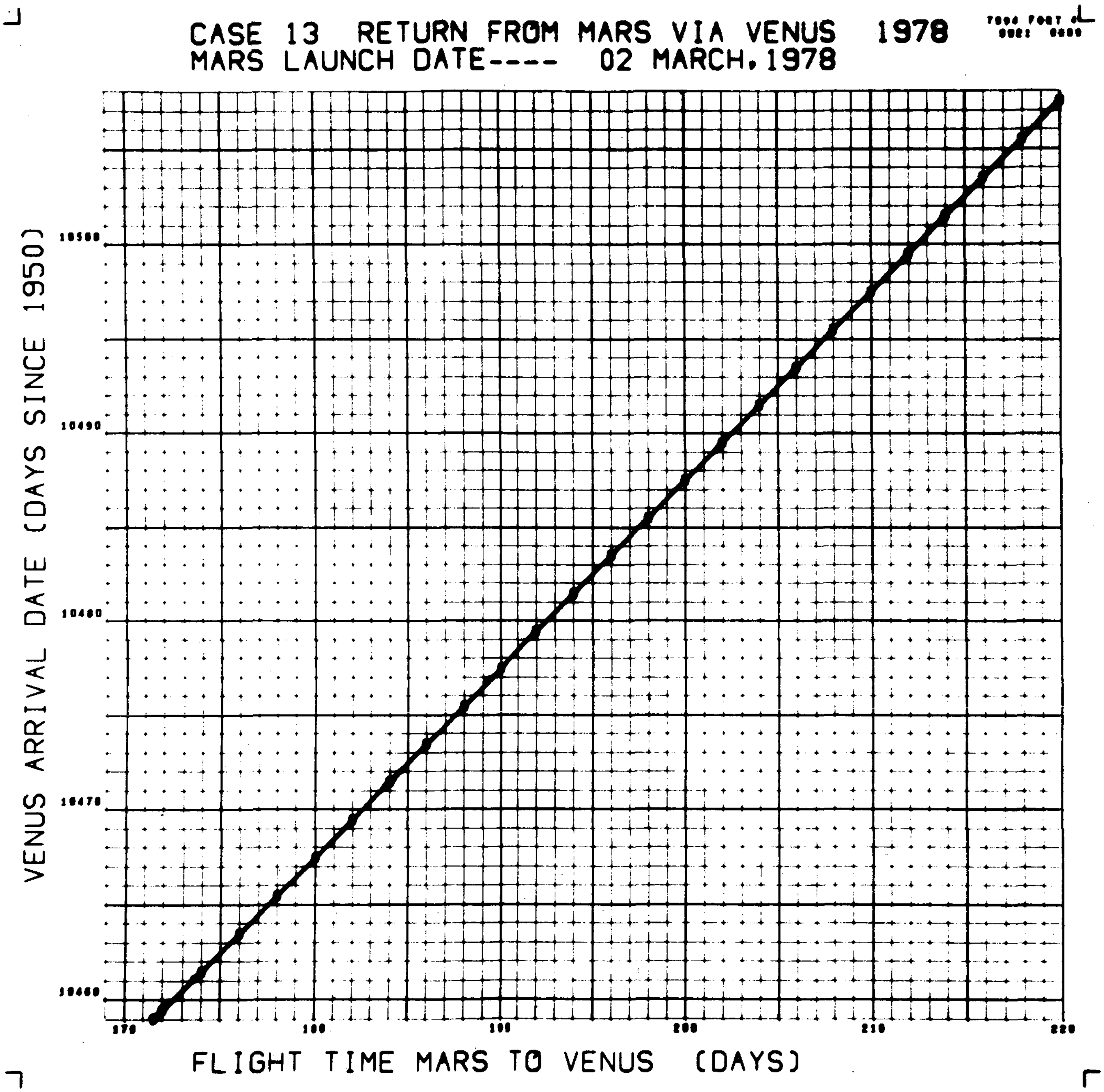
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

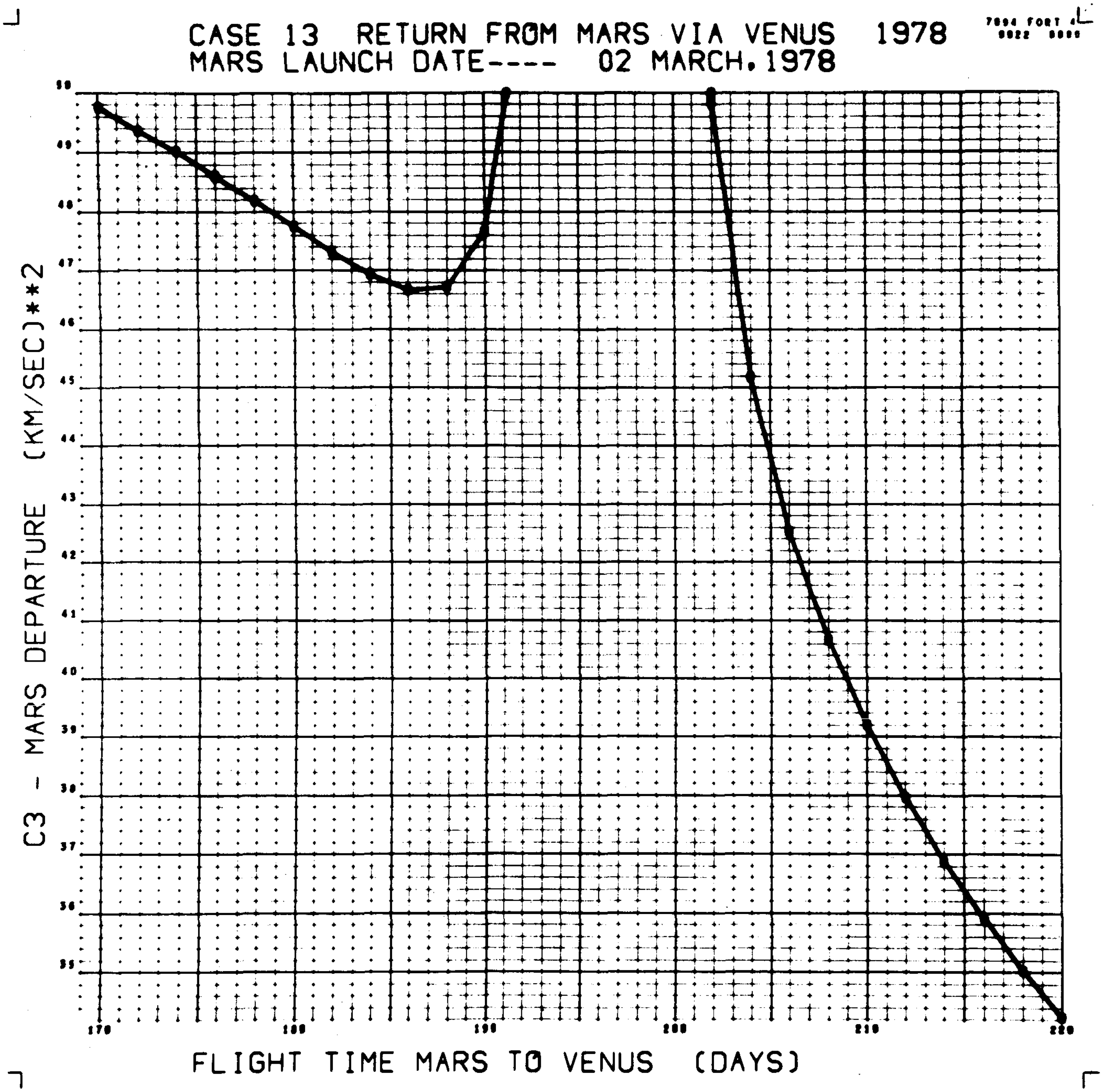
PLOTTING SYMBOL DEFINITIONS

PARAMETER FOR CURVES

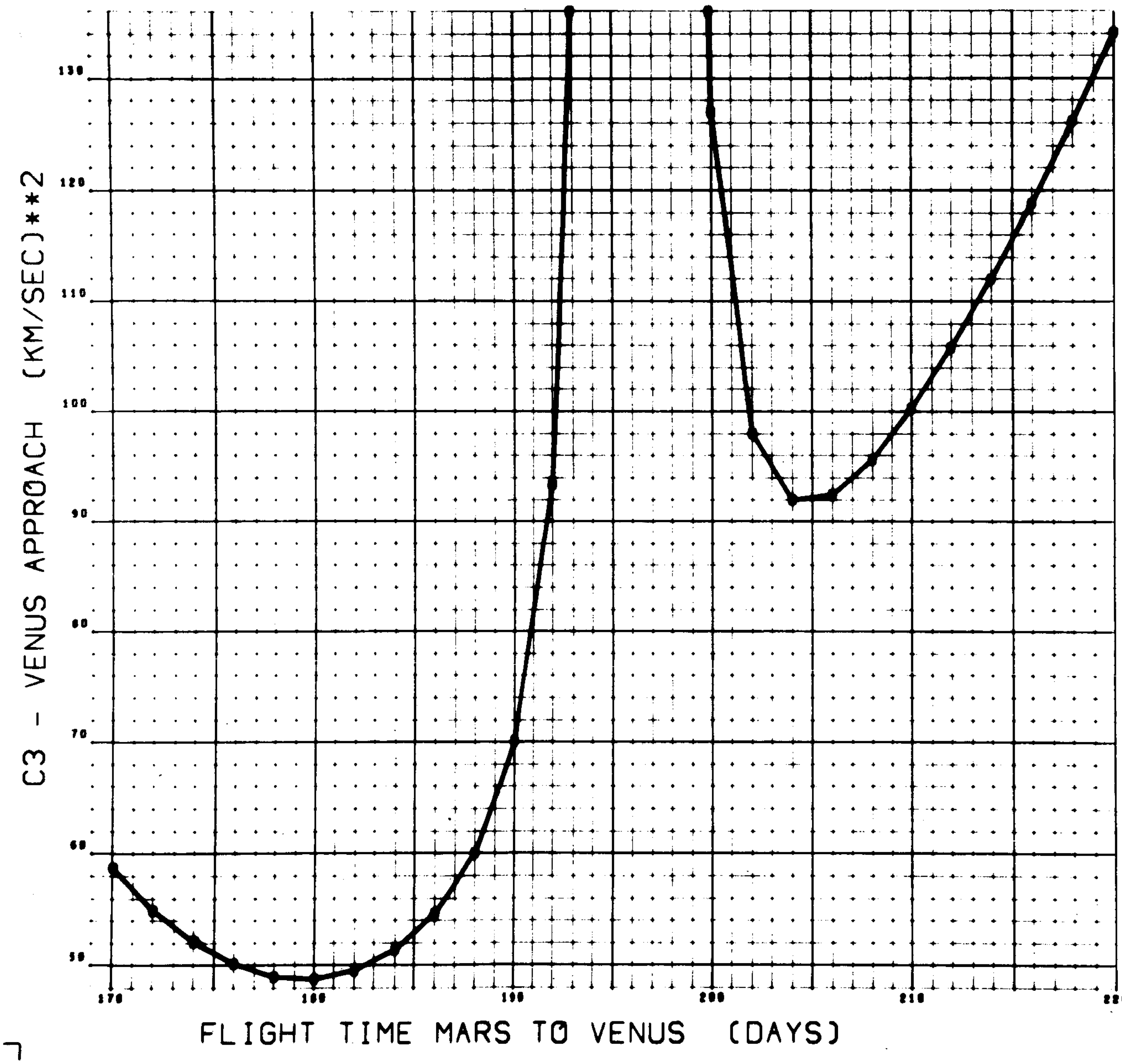
FLIGHT TIME (MARS TO EARTH) (DAYS)

SYMBOL	PARAMETER VALUE
■	$2.7000 \times 10^{+02}$

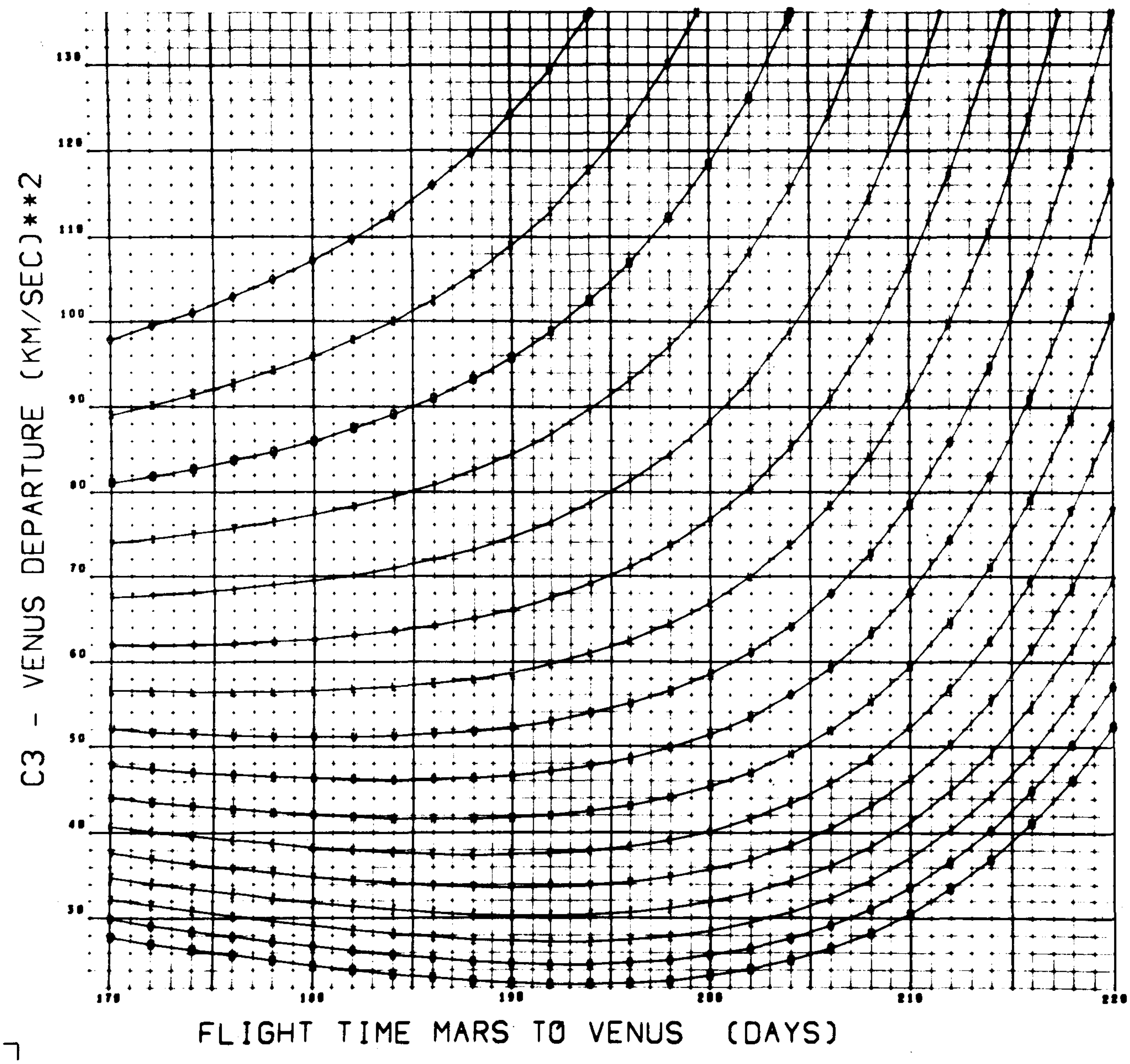

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
VENUS ARRIVAL DATE (DAYS SINCE 1950)
FLIGHT TIME MARS TO VENUS (DAYS)

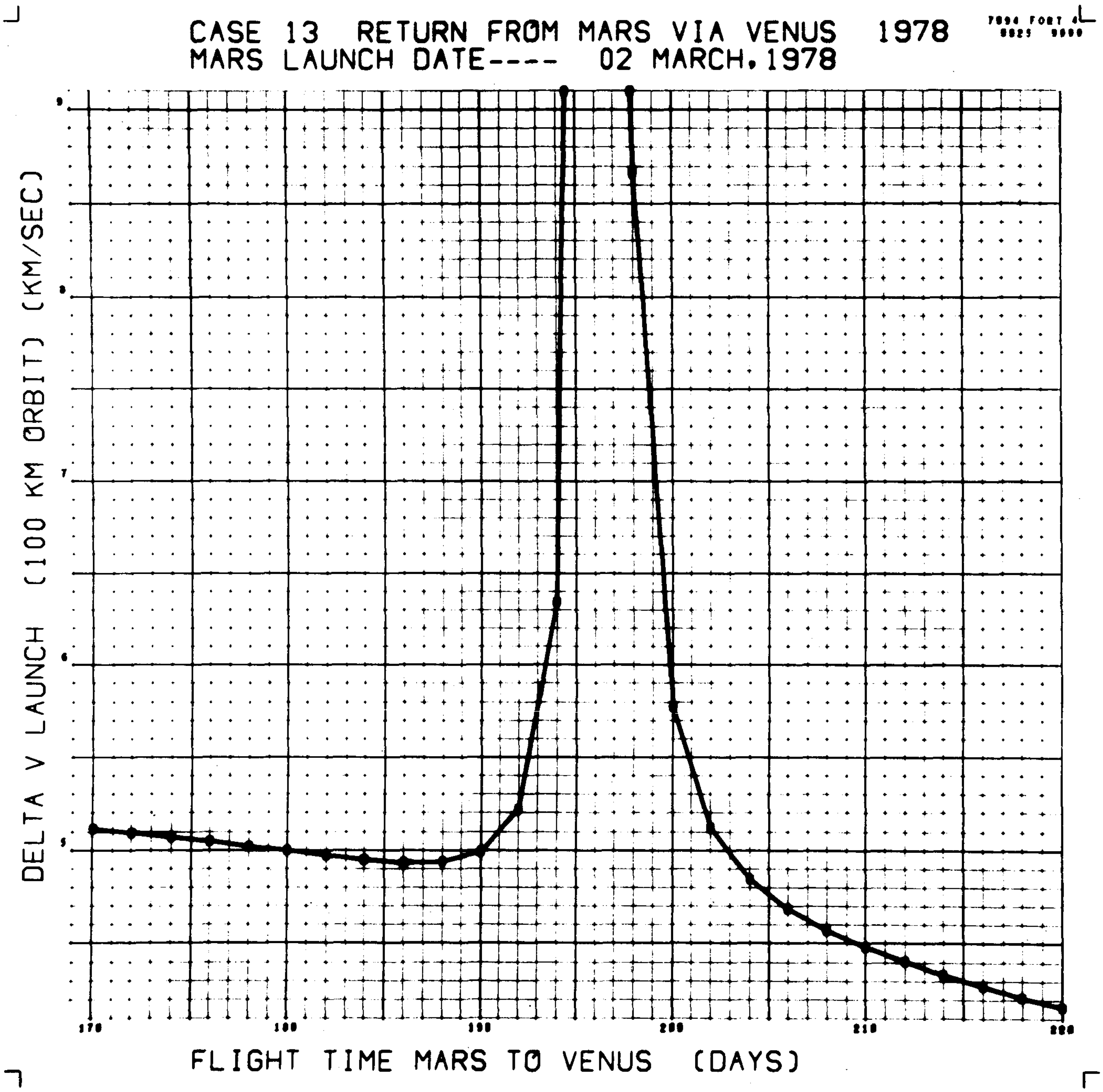

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH,1978
C3 - MARS DEPARTURE (KM/SEC)**2
FLIGHT TIME MARS TO VENUS (DAYS)

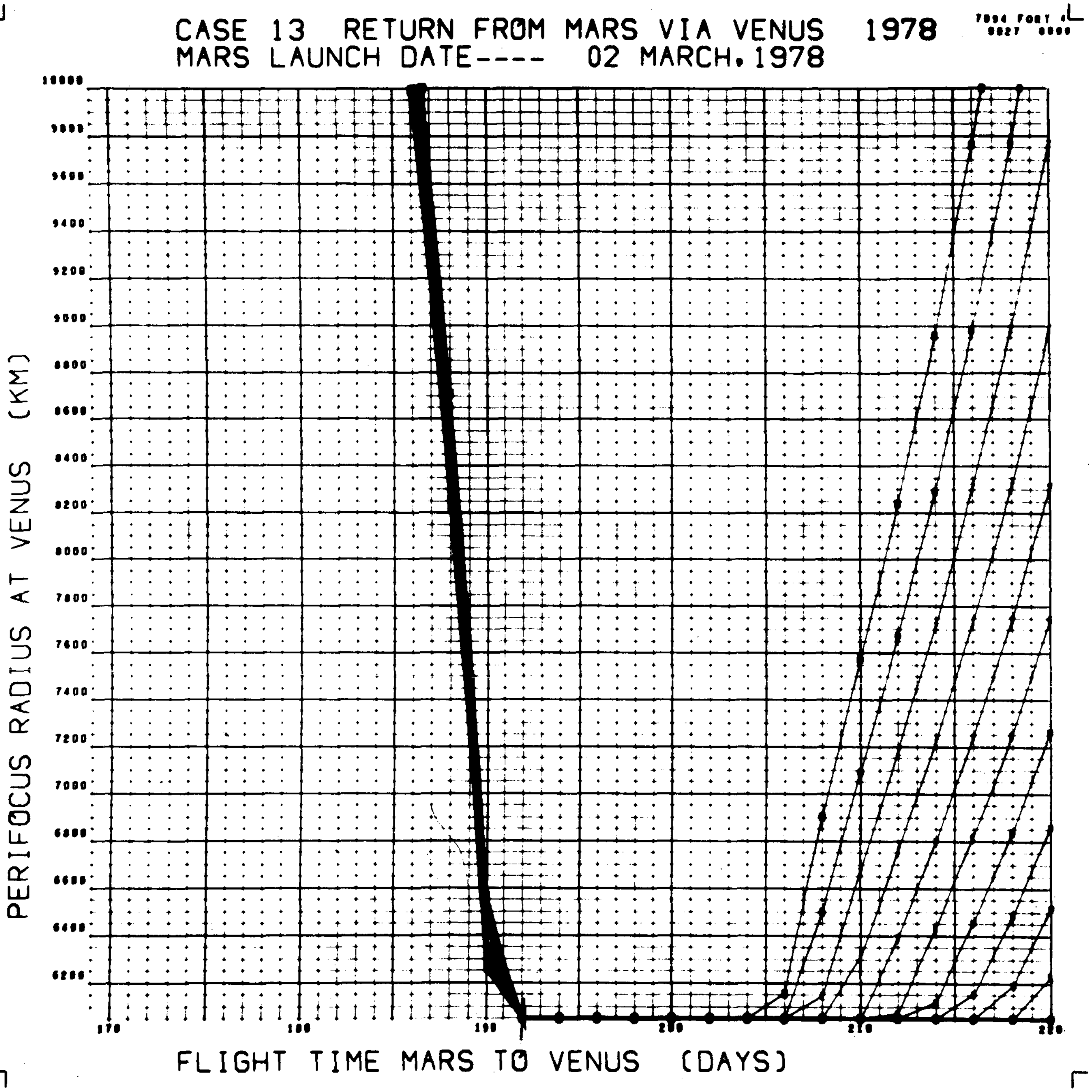
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

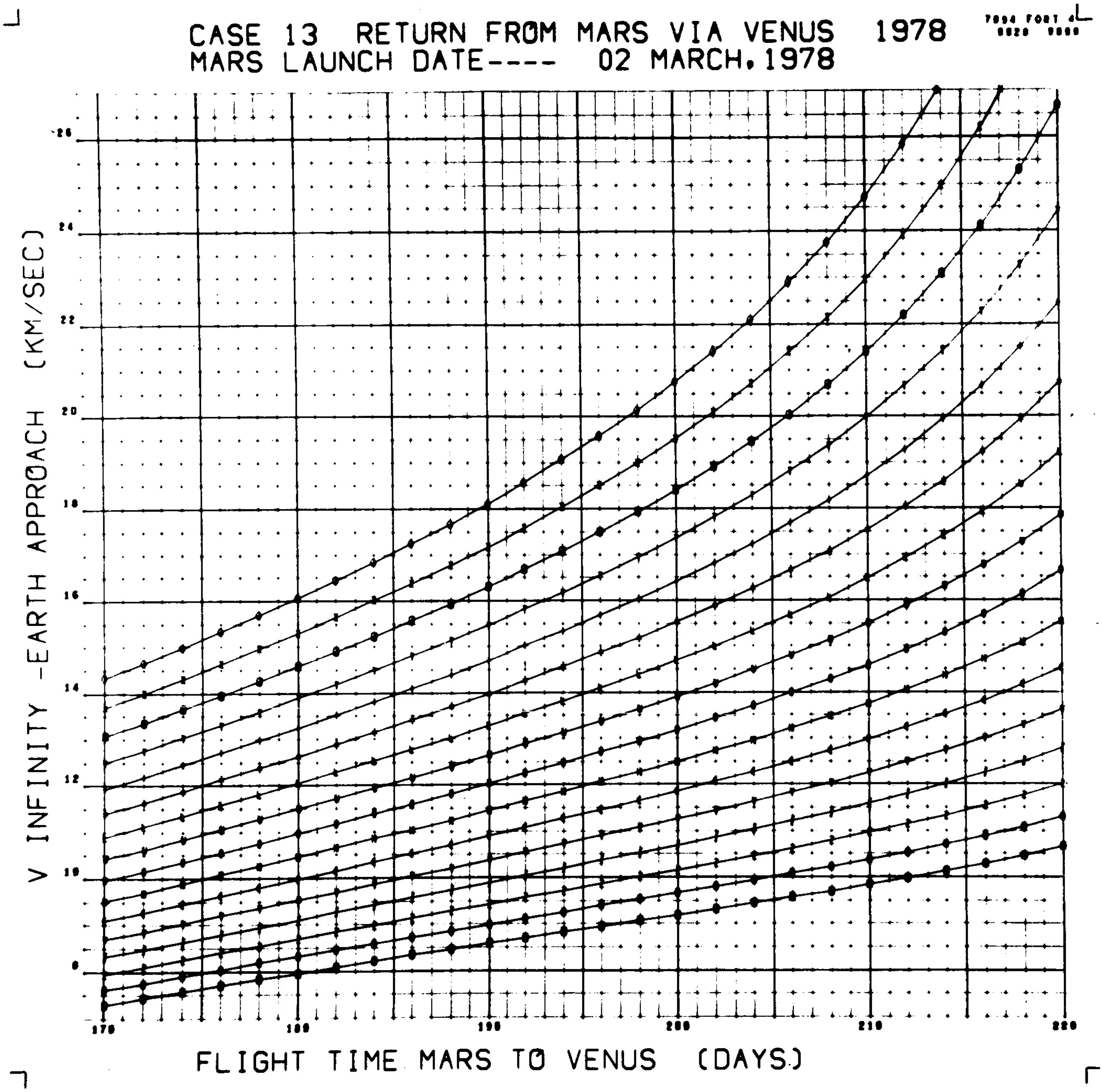


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

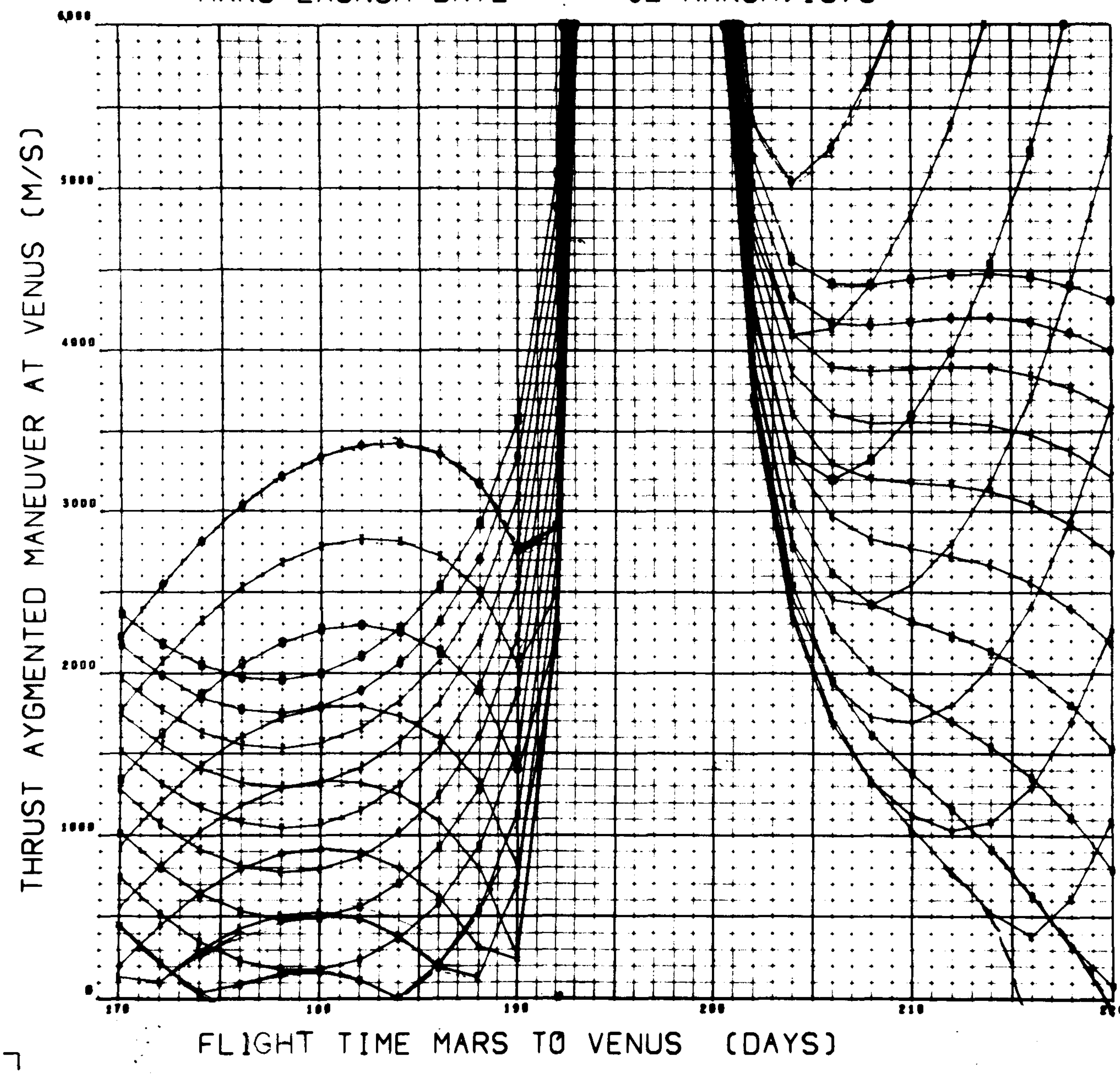



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
DELTA V LAUNCH (100 KM ORBIT) (KM/SEC)
FLIGHT TIME MARS TO VENUS (DAYS)

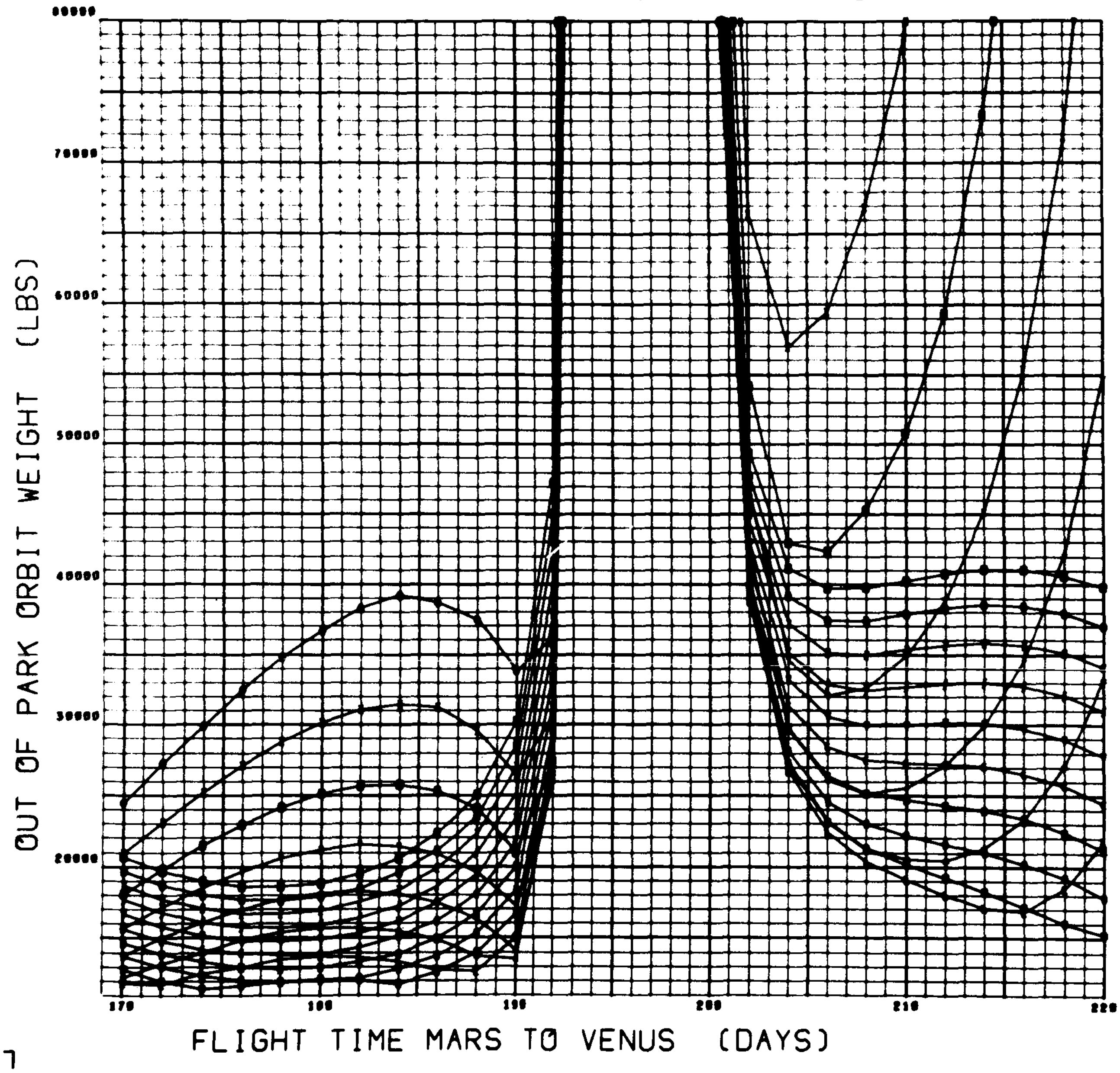

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
PERIFOCUS RADIUS AT VENUS (KM)
FLIGHT TIME MARS TO VENUS (DAYS)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
V INFINITY -EARTH APPROACH (KM/SEC)
26
24
22
20
18
16
14
12
10
8
170
180
190
200
210
220
FLIGHT TIME MARS TO VENUS (DAYS)

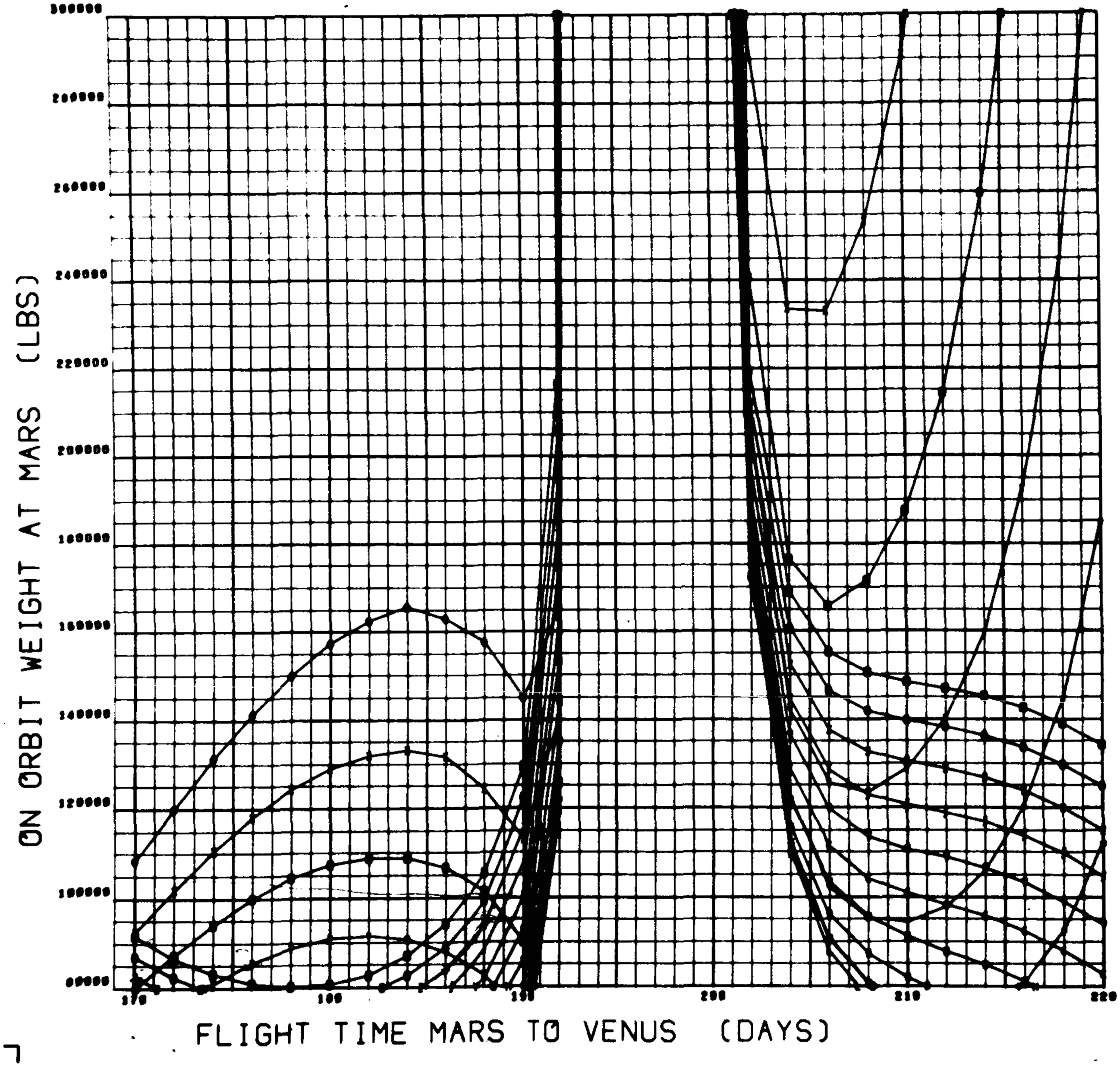
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



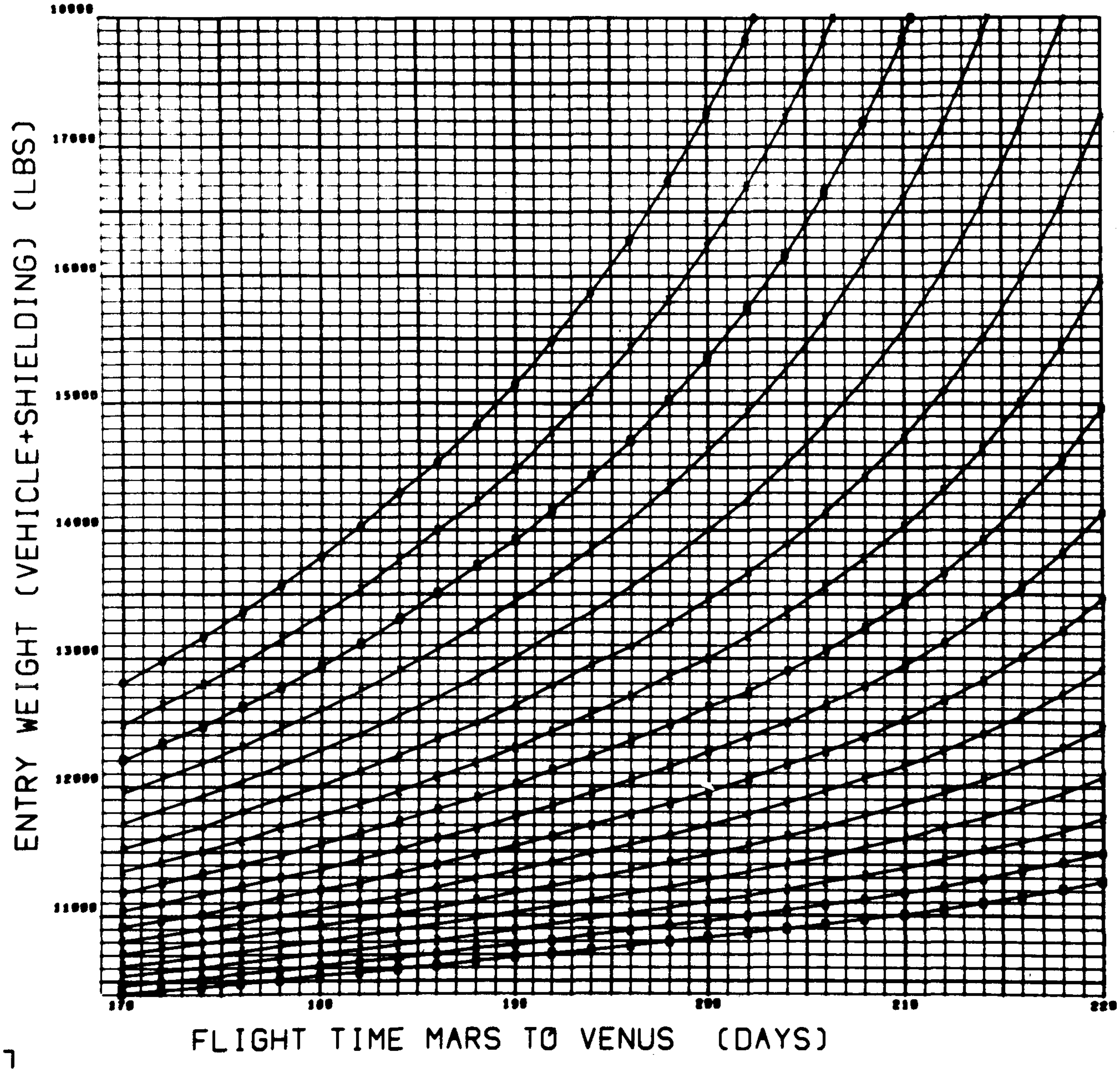
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



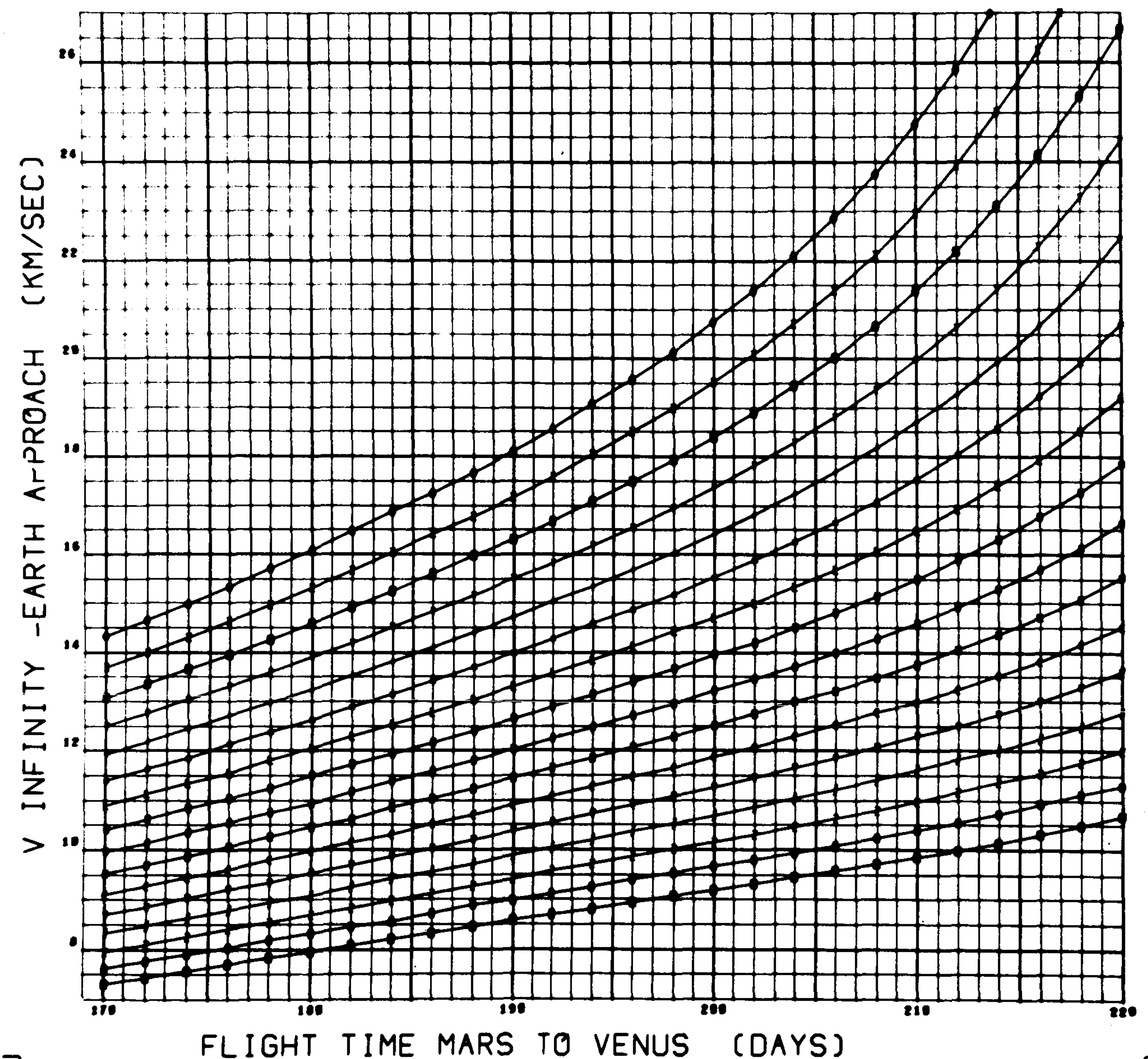
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



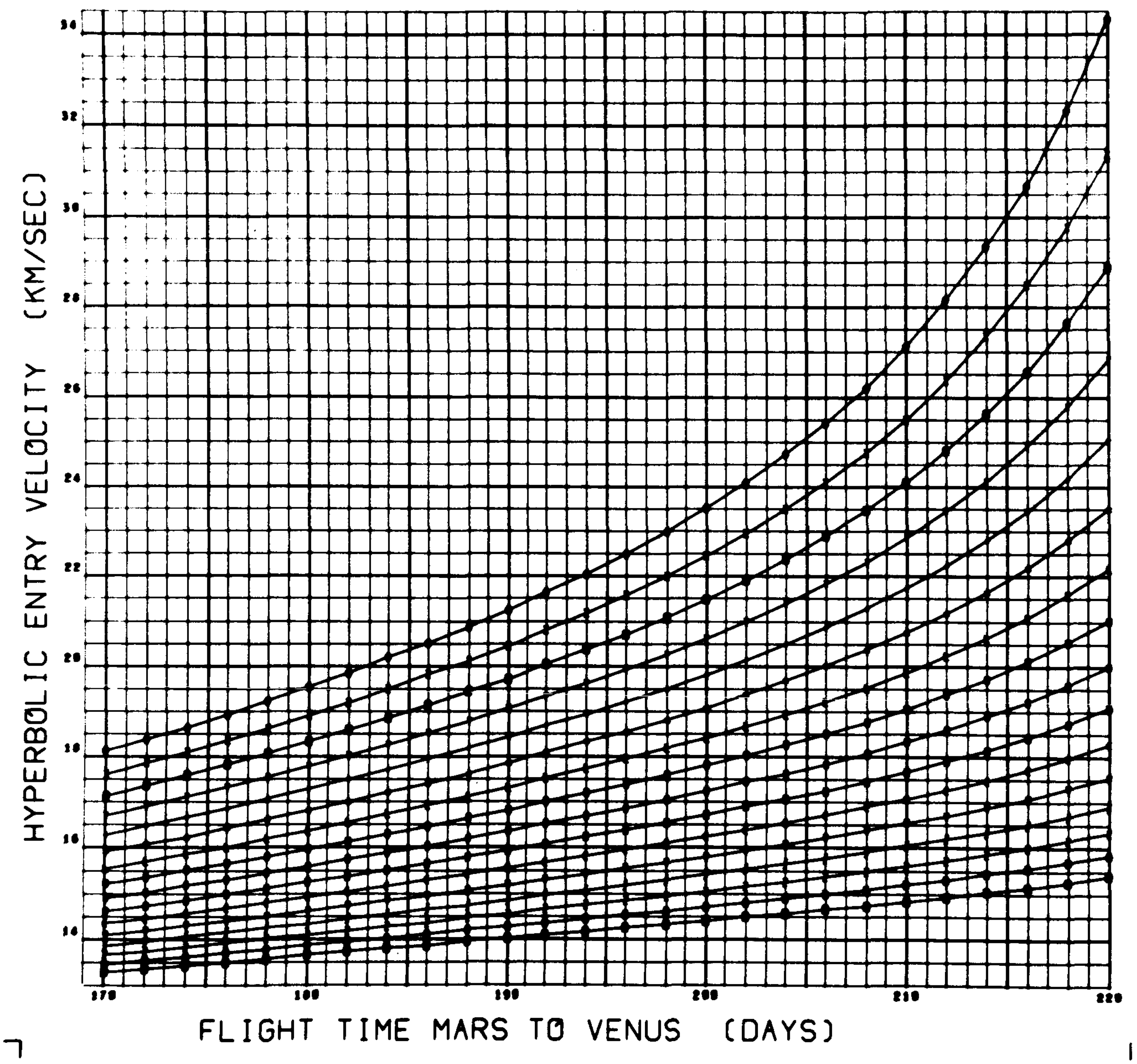
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

PLOTTING SYMBOL DEFINITIONS

PARAMETER FOR CURVES
FLIGHT TIME - MARS TO VENUS (DAYS)

SYMBOL	PARAMETER VALUE
O	$1.7000 \times 10^{+02}$
X	$1.7200 \times 10^{+02}$
□	$1.7400 \times 10^{+02}$
Y	$1.7600 \times 10^{+02}$
+	$1.7800 \times 10^{+02}$
*	$1.8000 \times 10^{+02}$
L	$1.8200 \times 10^{+02}$
U	$1.8400 \times 10^{+02}$
D	$1.8600 \times 10^{+02}$
H	$1.8800 \times 10^{+02}$
C	$1.9000 \times 10^{+02}$
V	$1.9200 \times 10^{+02}$
J	$1.9400 \times 10^{+02}$
Z	$1.9600 \times 10^{+02}$
●	$1.9800 \times 10^{+02}$

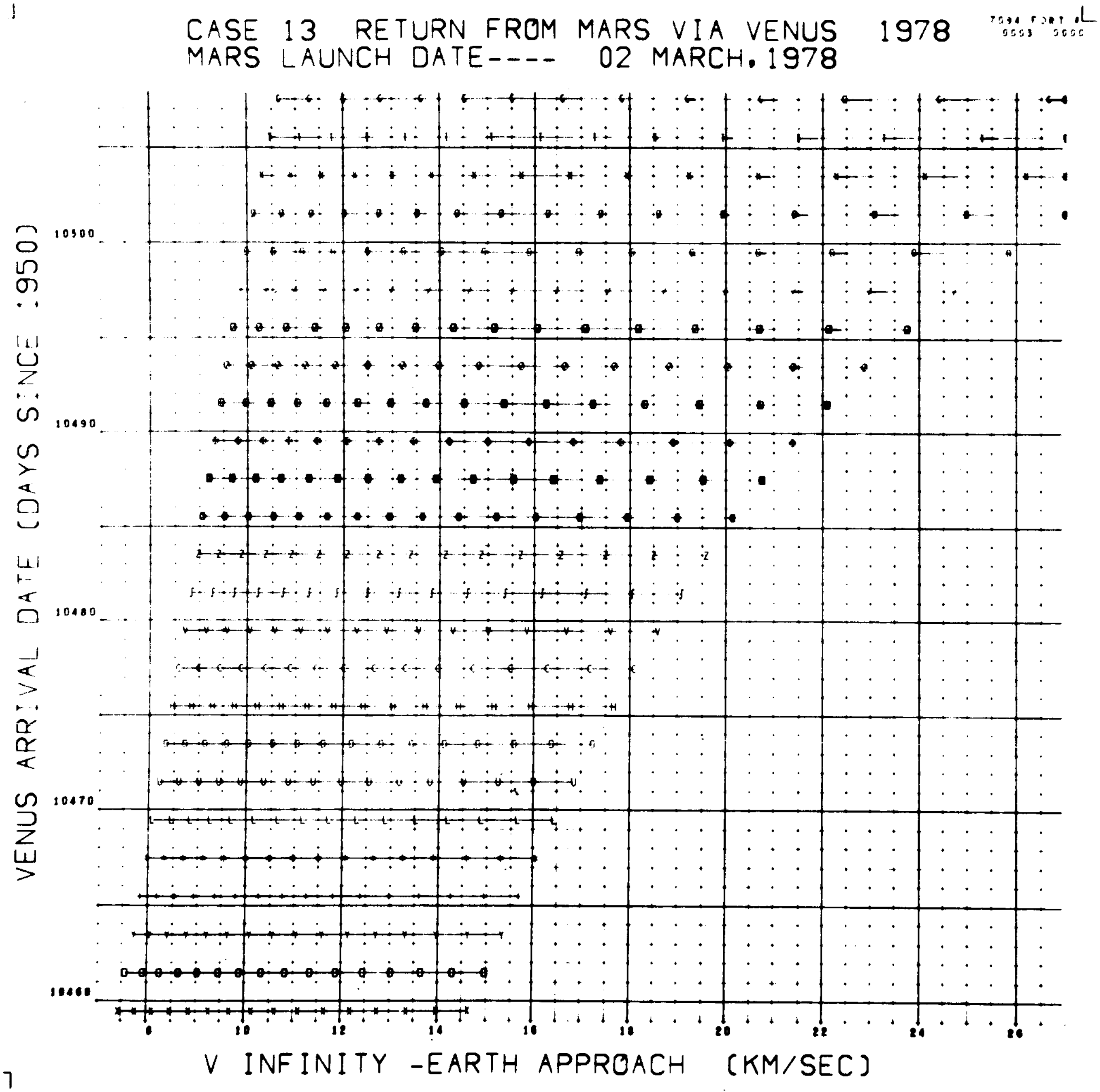
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

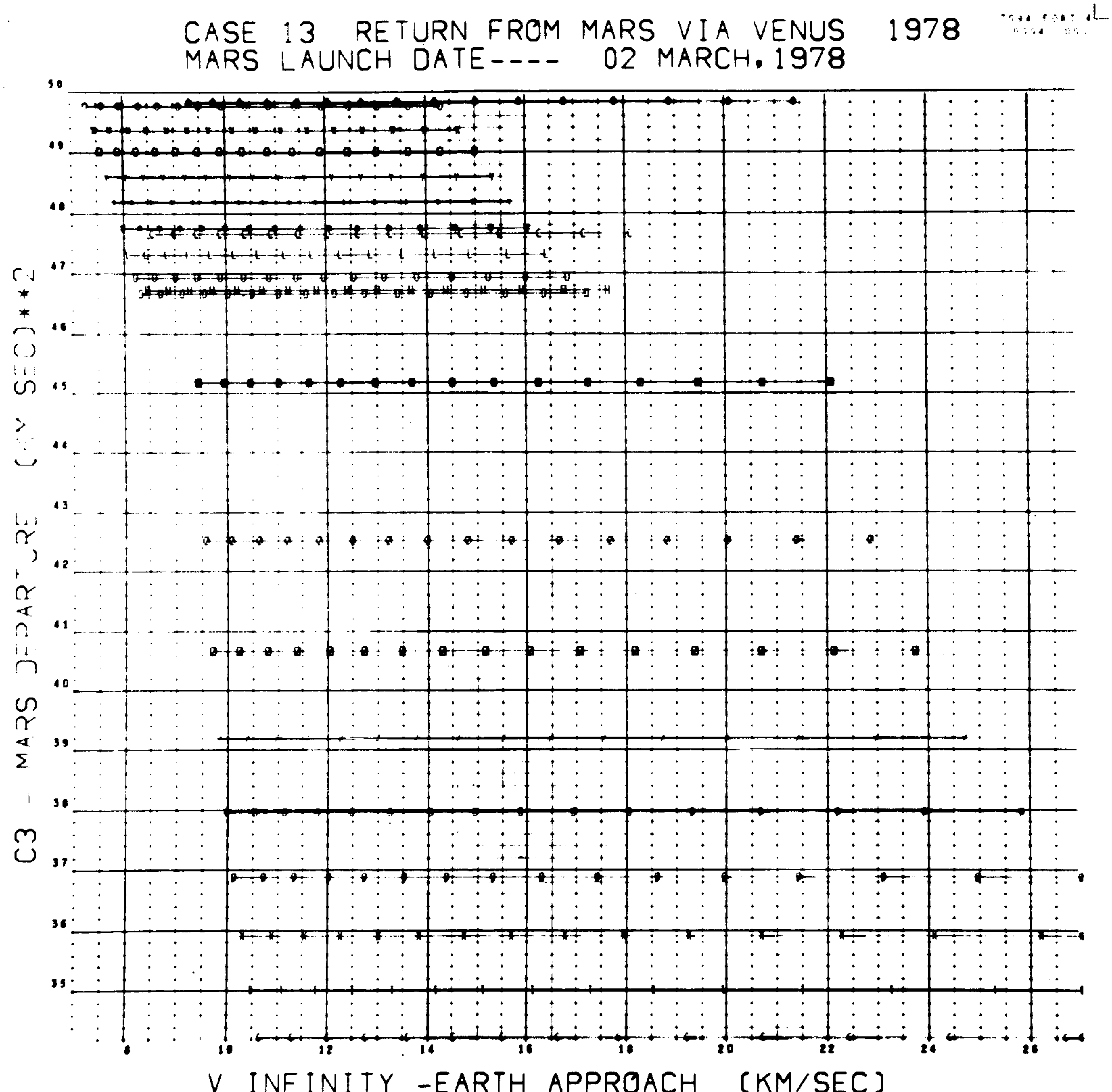
PLOTTING SYMBOL DEFINITIONS

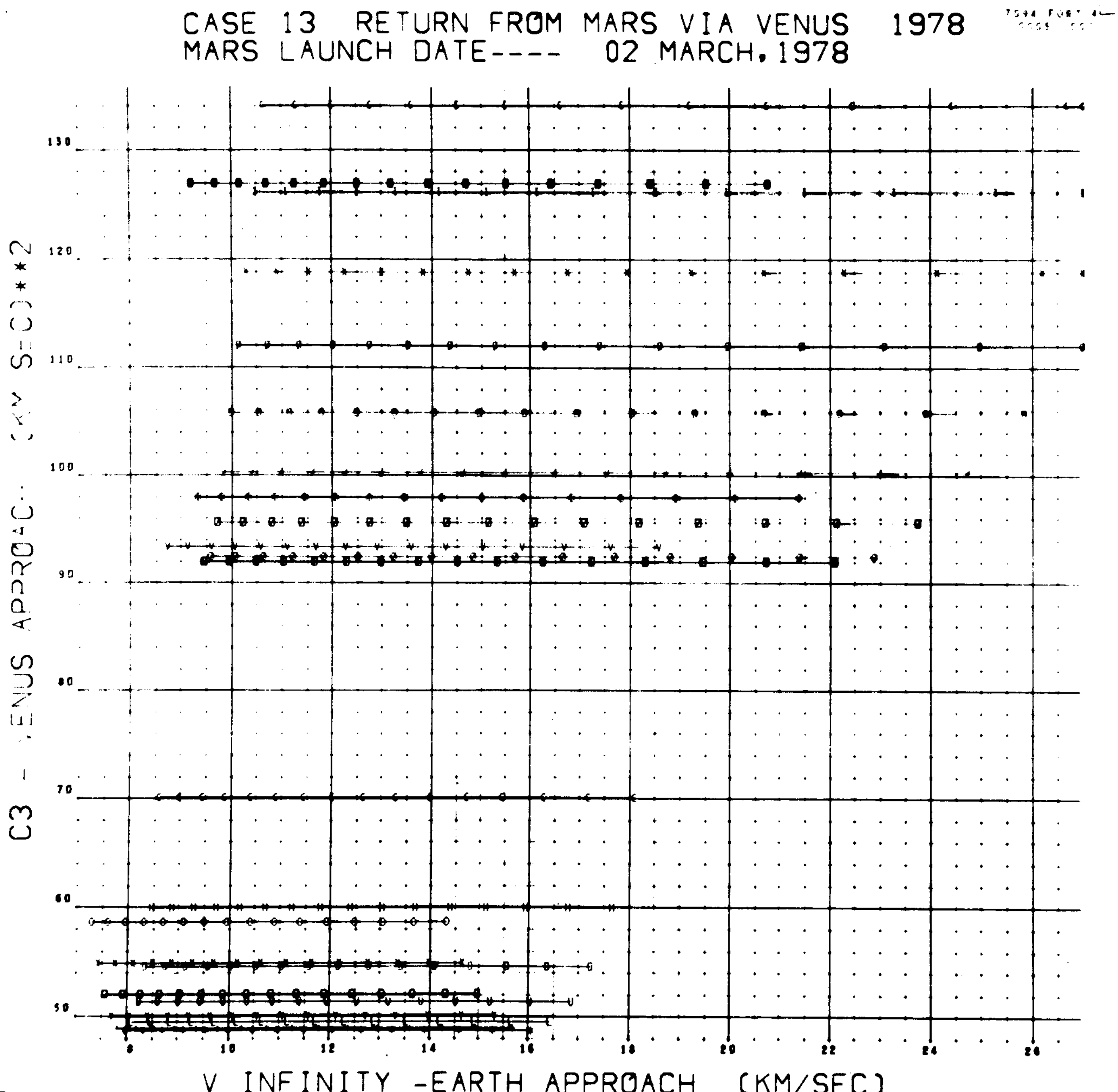
PARAMETER FOR CURVES

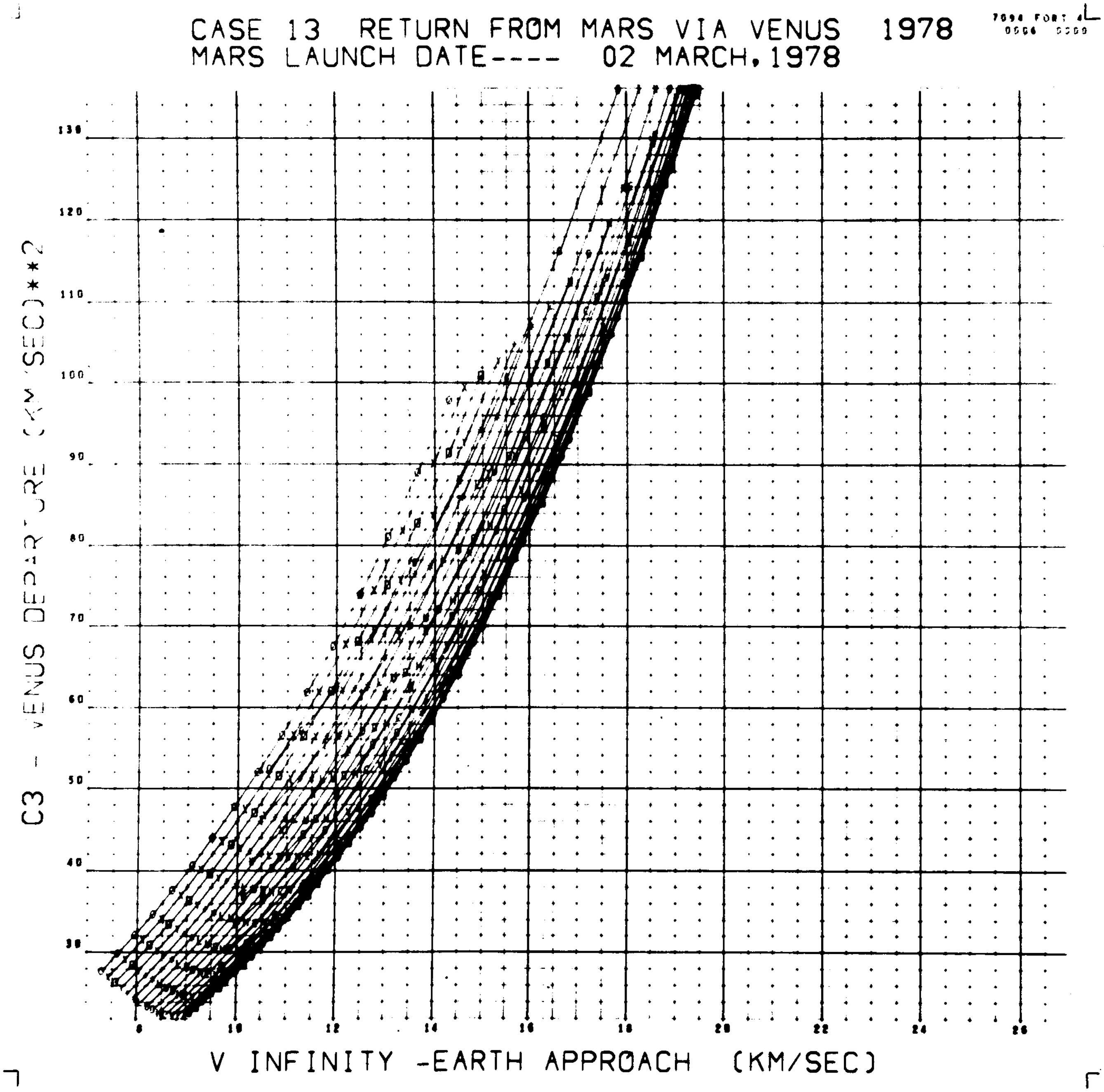
FLIGHT TIME - MARS TO VENUS (DAYS

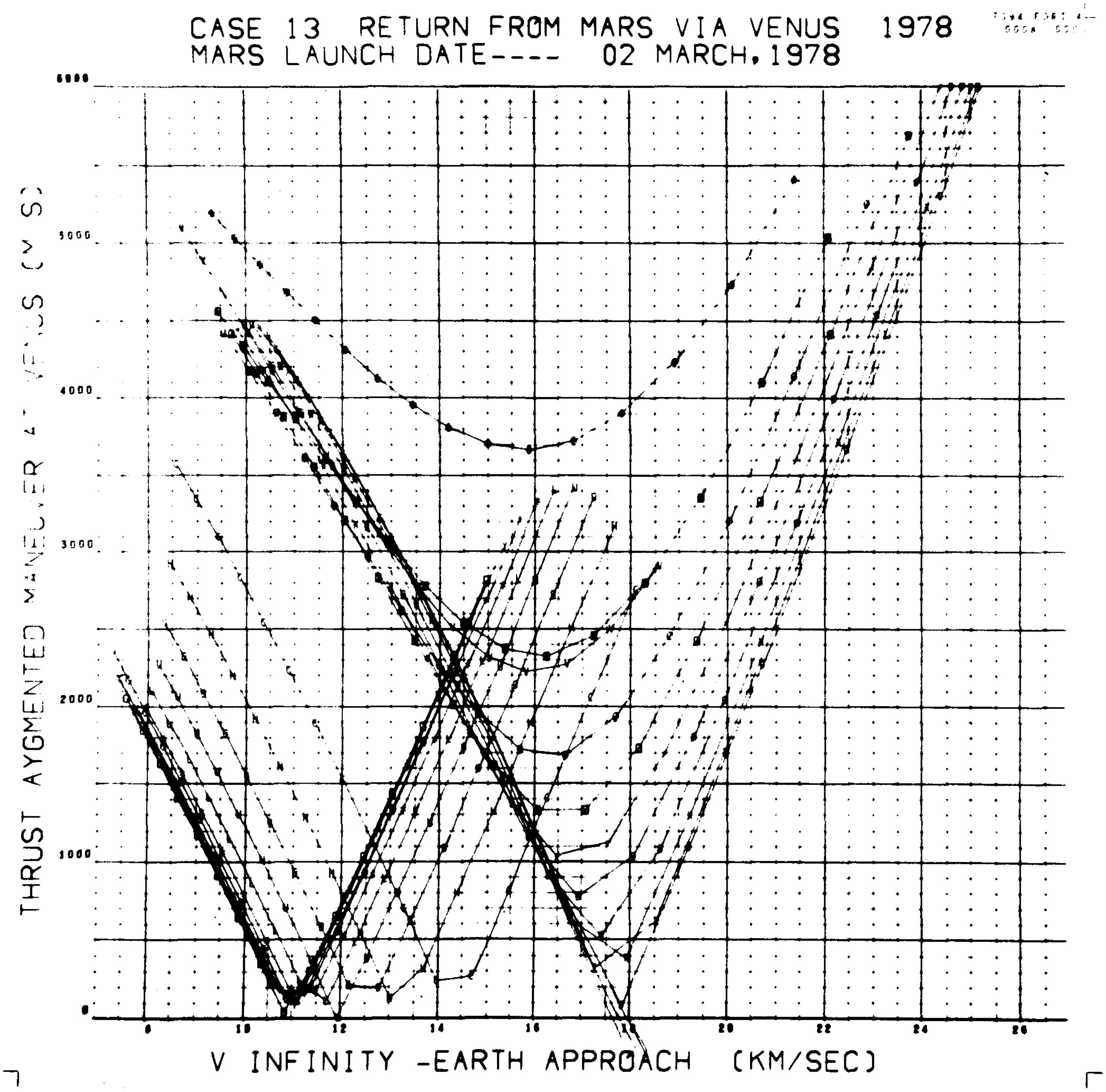
SYMBOL	PARAMETER VALUE
■	$2.0000 \times 10^{+02}$
●	$2.0200 \times 10^{+02}$
■	$2.0400 \times 10^{+02}$
@	$2.0600 \times 10^{+02}$
Ø	$2.0800 \times 10^{+02}$
+	$2.1000 \times 10^{+02}$
9	$2.1200 \times 10^{+02}$
9	$2.1400 \times 10^{+02}$
k	$2.1600 \times 10^{+02}$
l	$2.1800 \times 10^{+02}$
.G	$2.2000 \times 10^{+02}$


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
VENUS ARRIVAL DATE (DAYS SINCE 1950)
10500
10490
10480
10470
V INFINITY -EARTH APPROACH (KM/SEC)
8
10
12
14
16
18
20
22
24
26


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
C3 - MARS DEPARTURE (KM SEC)**2
V INFINITY -EARTH APPROACH (KM/SEC)
50
49
48
47
46
45
44
43
42
41
40
39
38
37
36
35
8
10
12
14
16
18
20
22
24
26

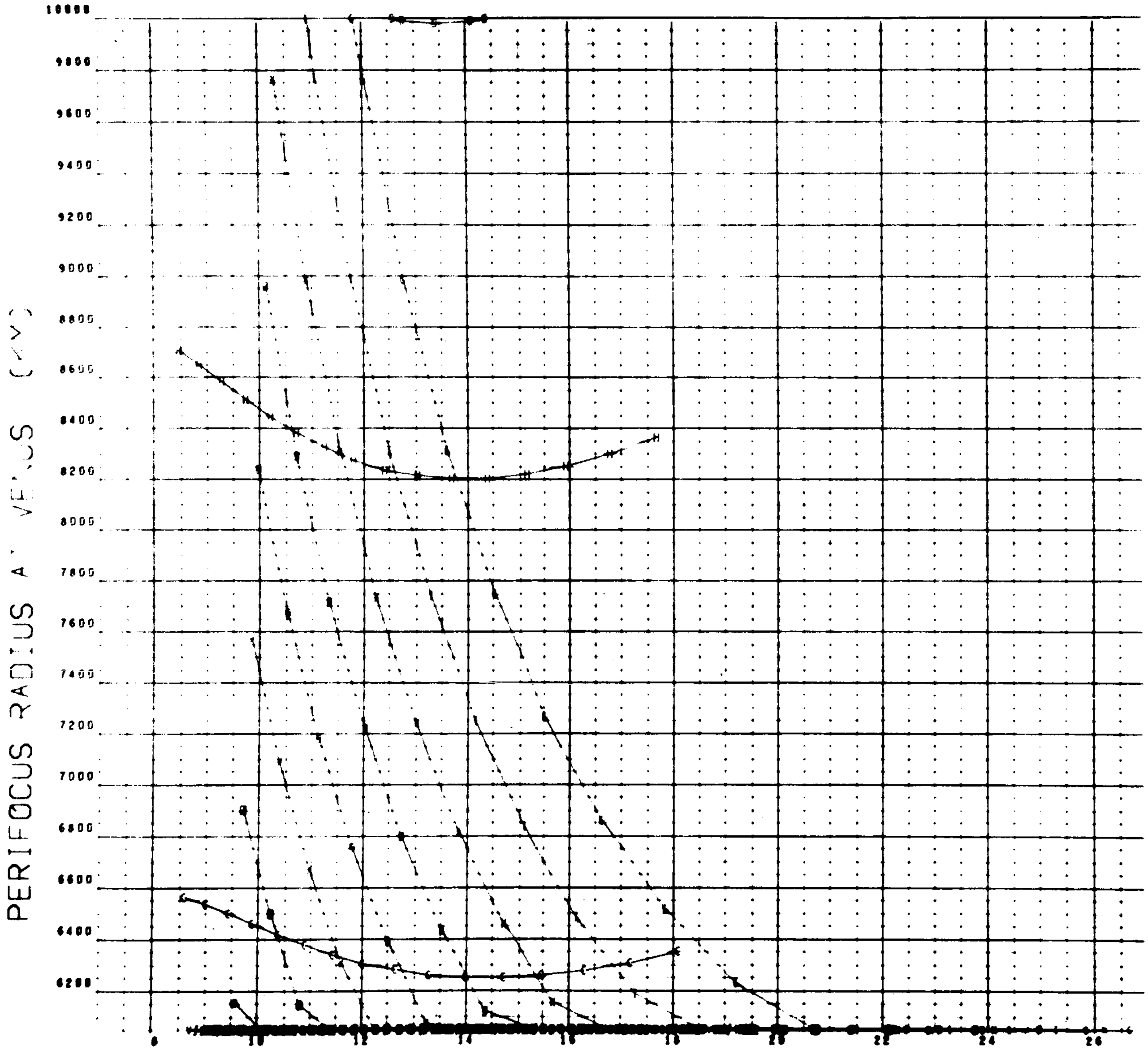

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH,1978
C3 - VENUS APPROAC. (KM SEC)**2
V INFINITY -EARTH APPROACH (KM/SEC)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH,1978
C3 - VENUS DEPARTURE (KM/SEC)**2
V INFINITY -EARTH APPROACH (KM/SEC)

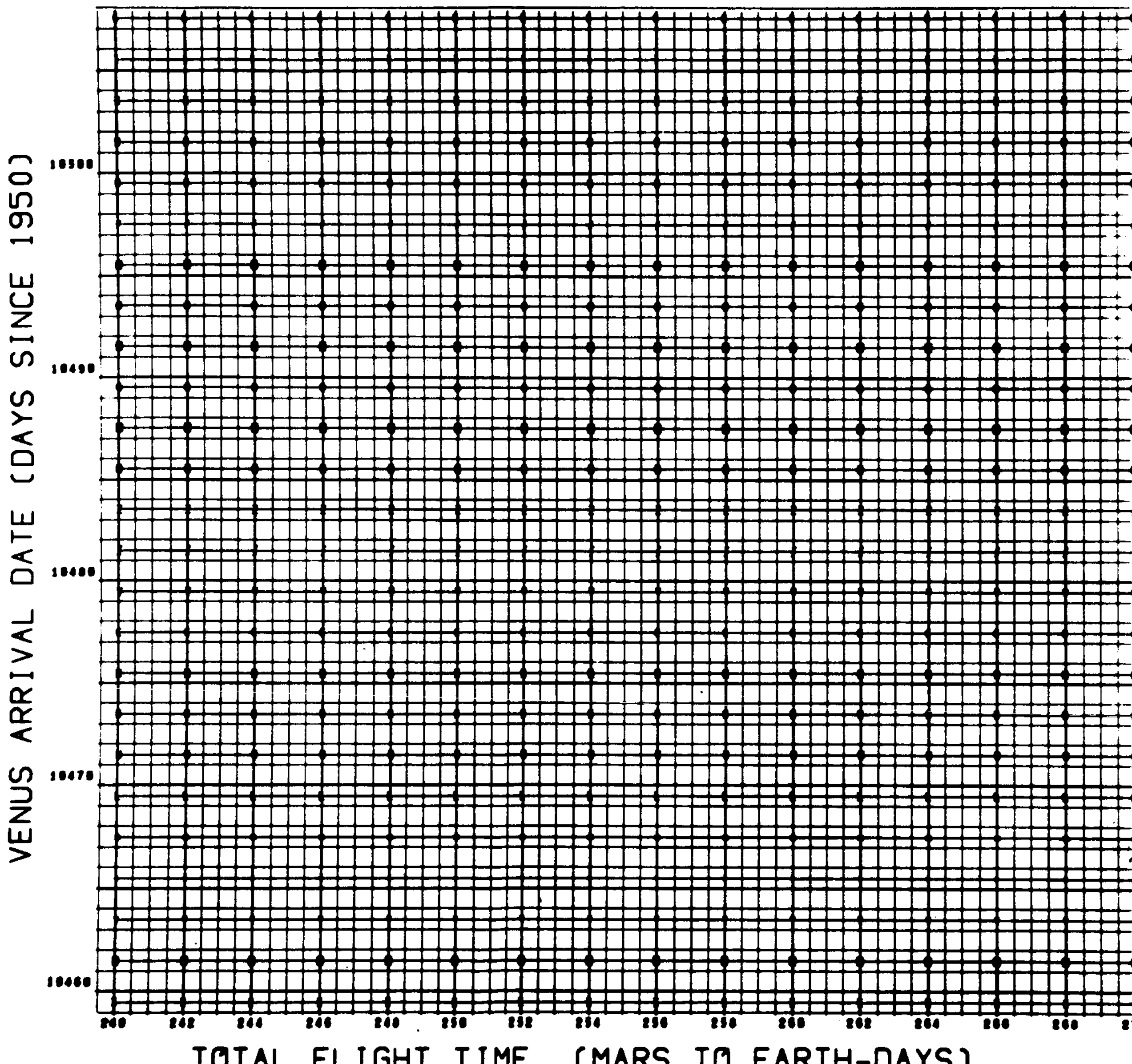


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
THRUST AYGMENTED MANEUVER AT VENUS (M/S)
5000
4000
3000
2000
1000
0
V INFINITY -EARTH APPROACH (KM/SEC)
10
12
14
16
18
20
22
24
26

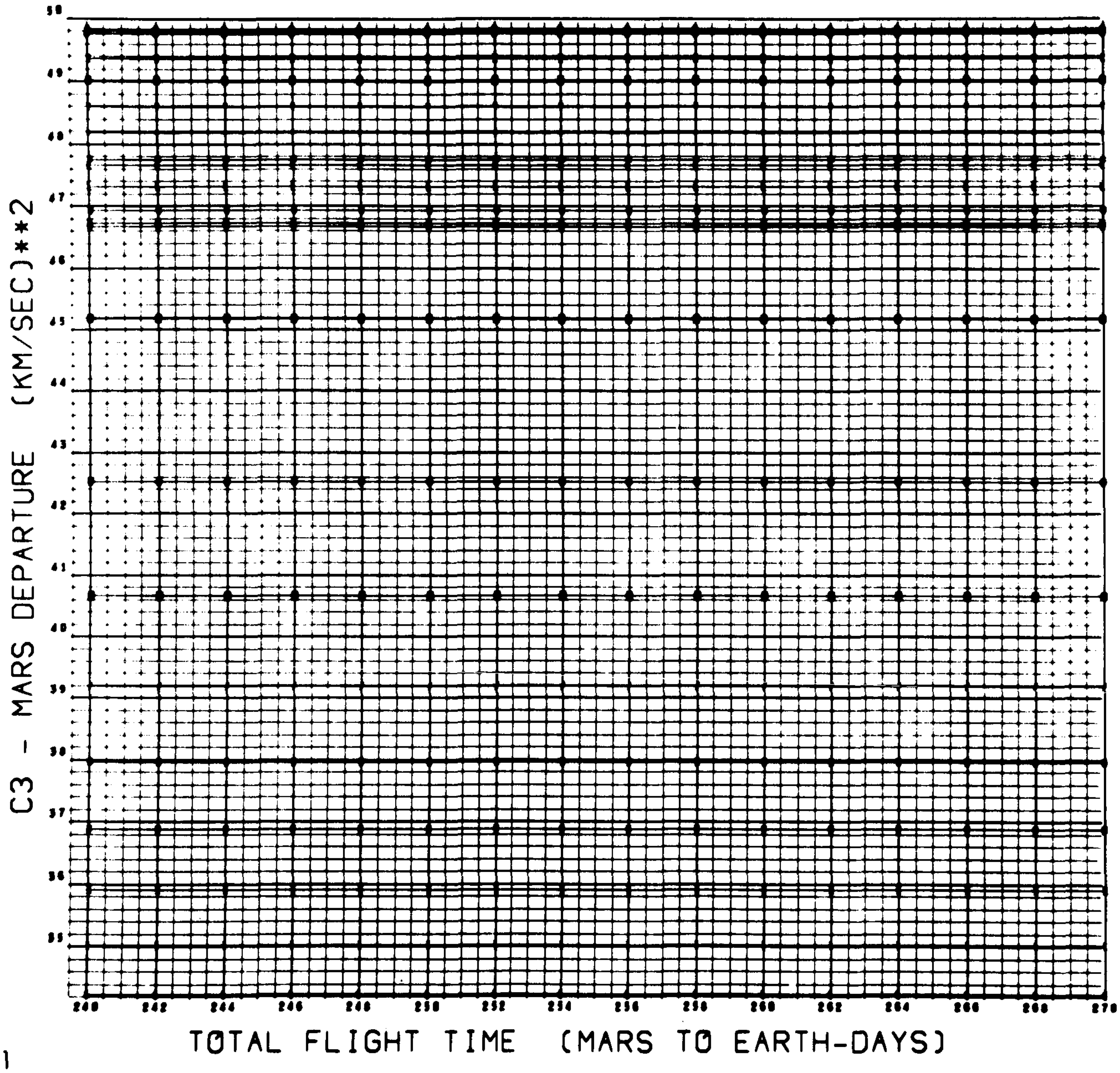
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



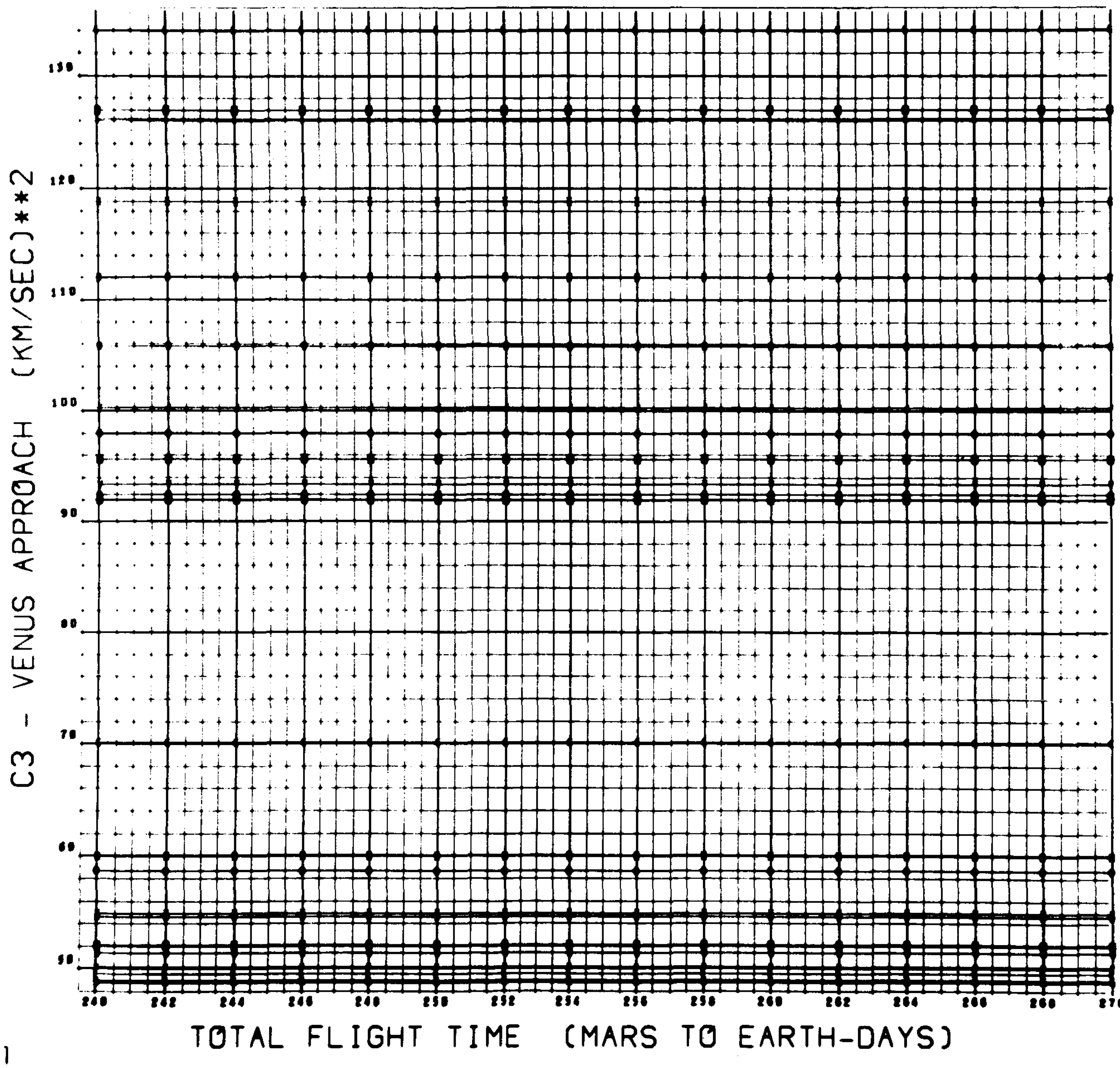
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978

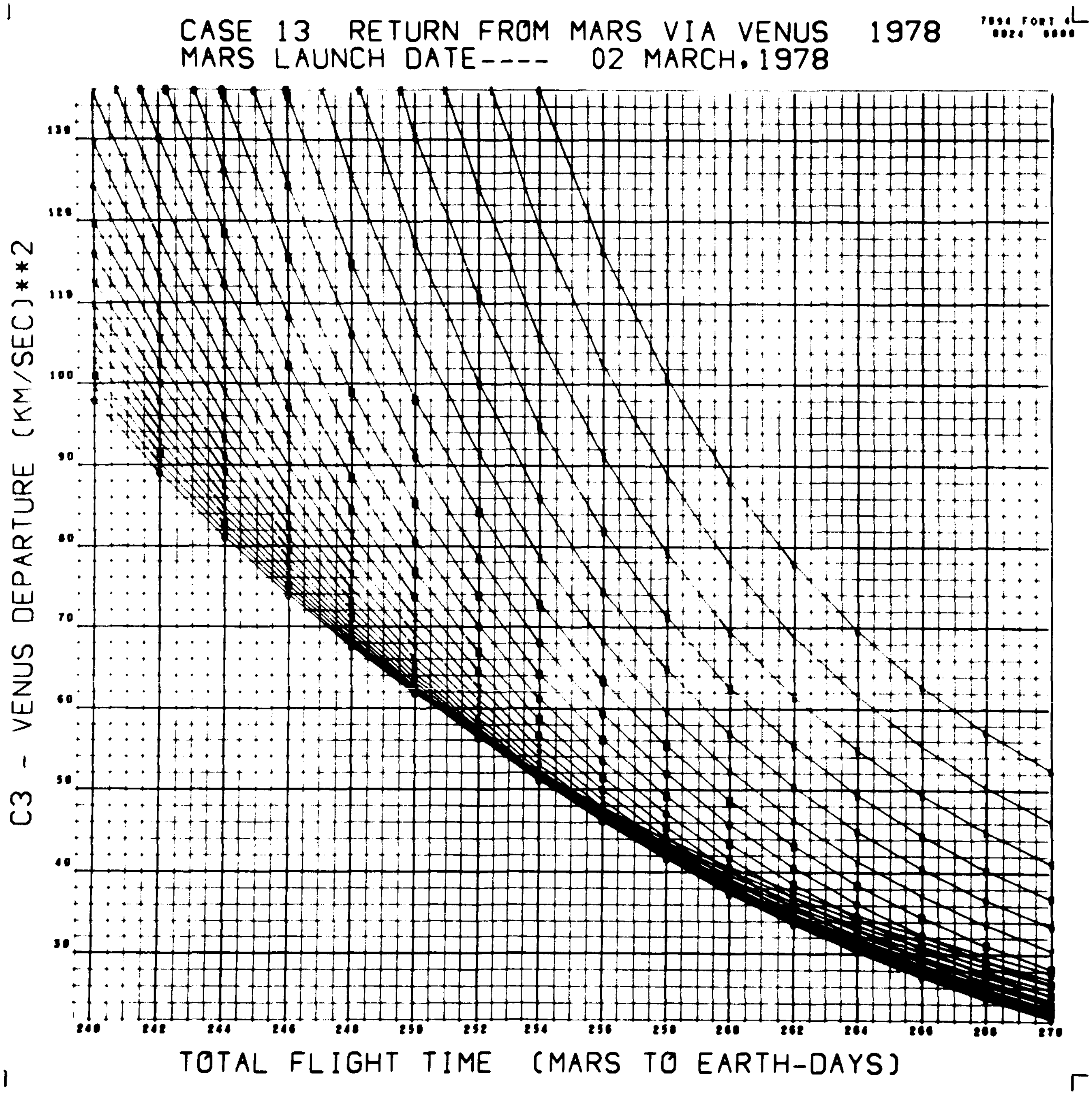


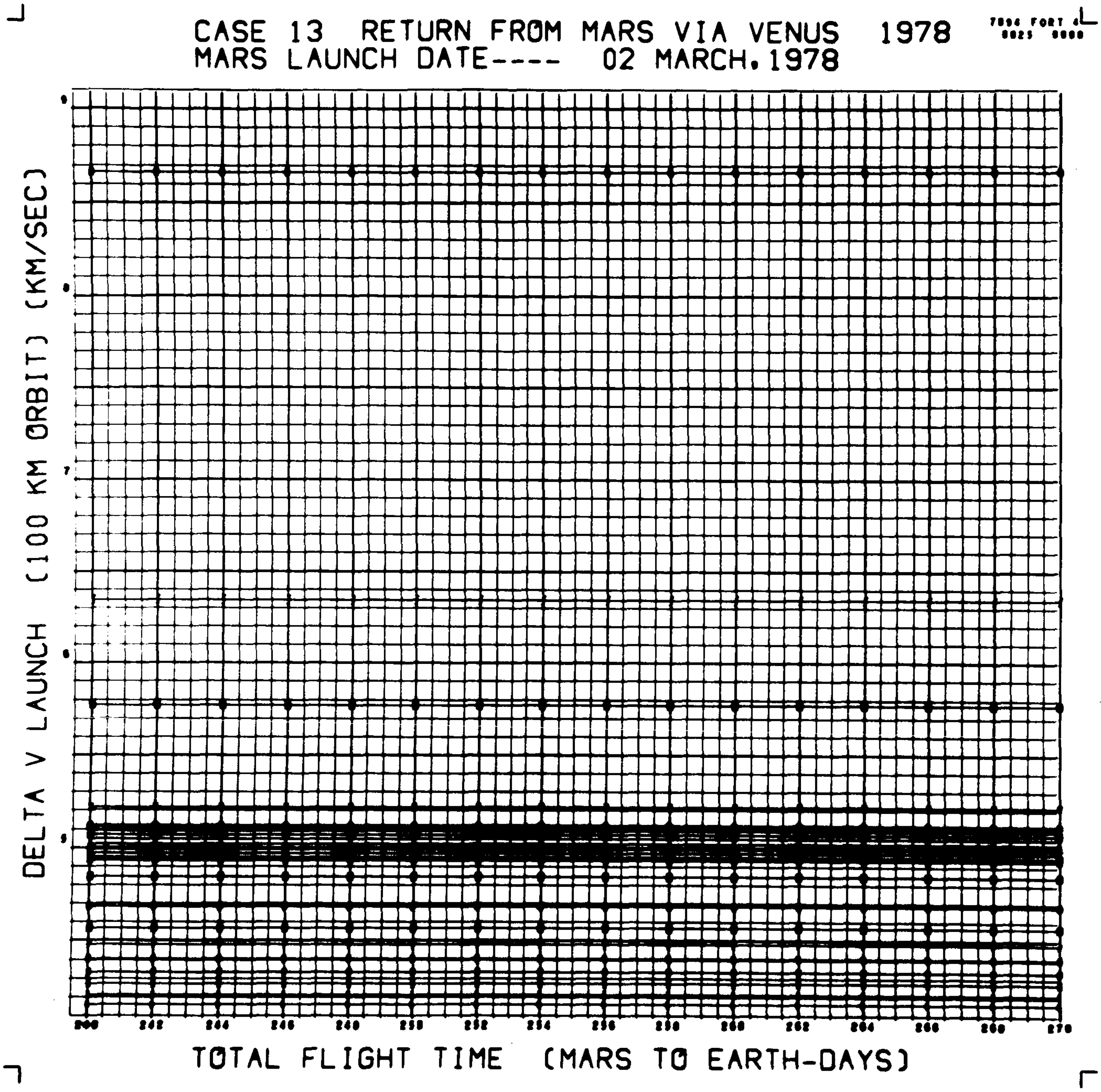
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



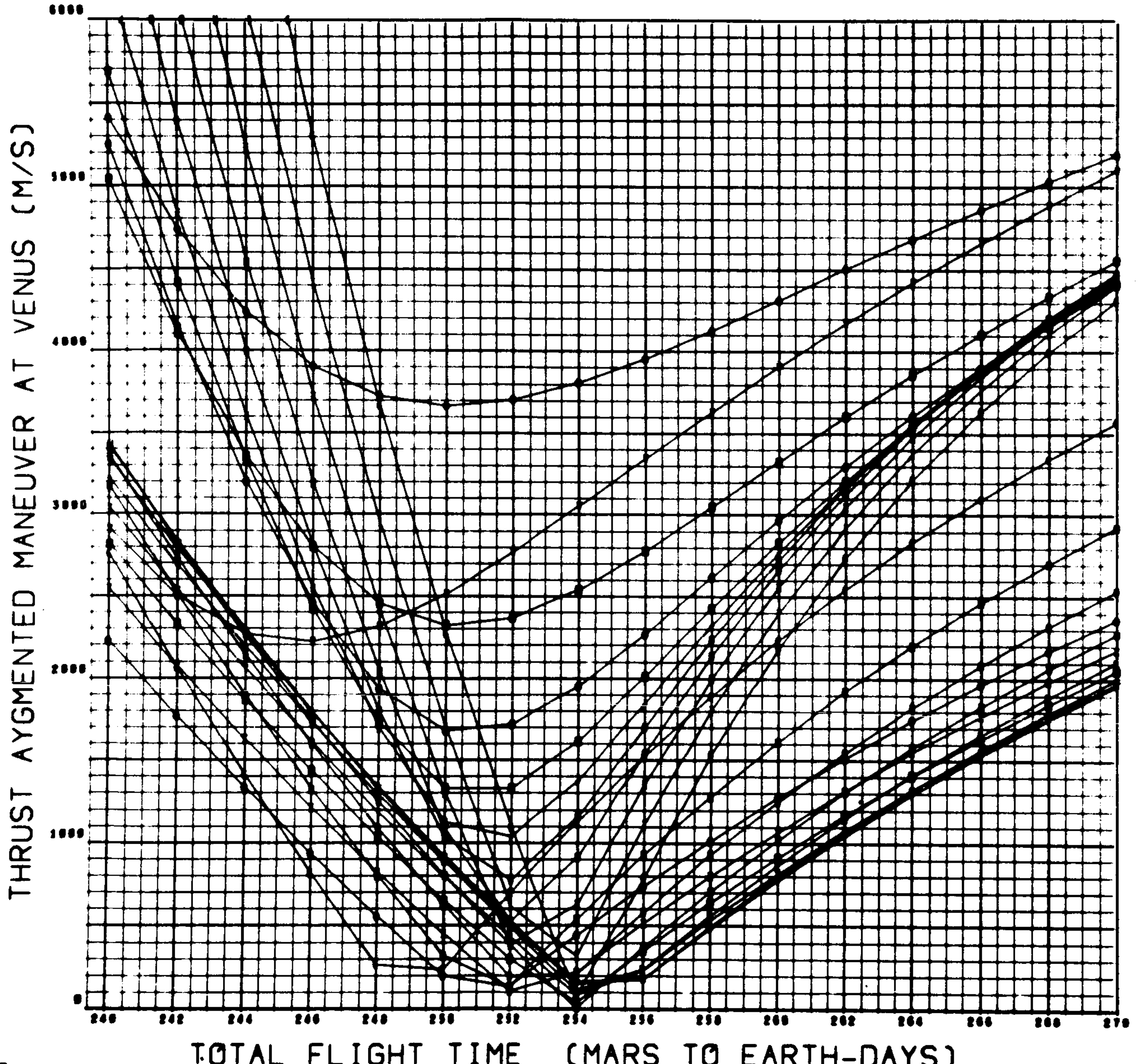
CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



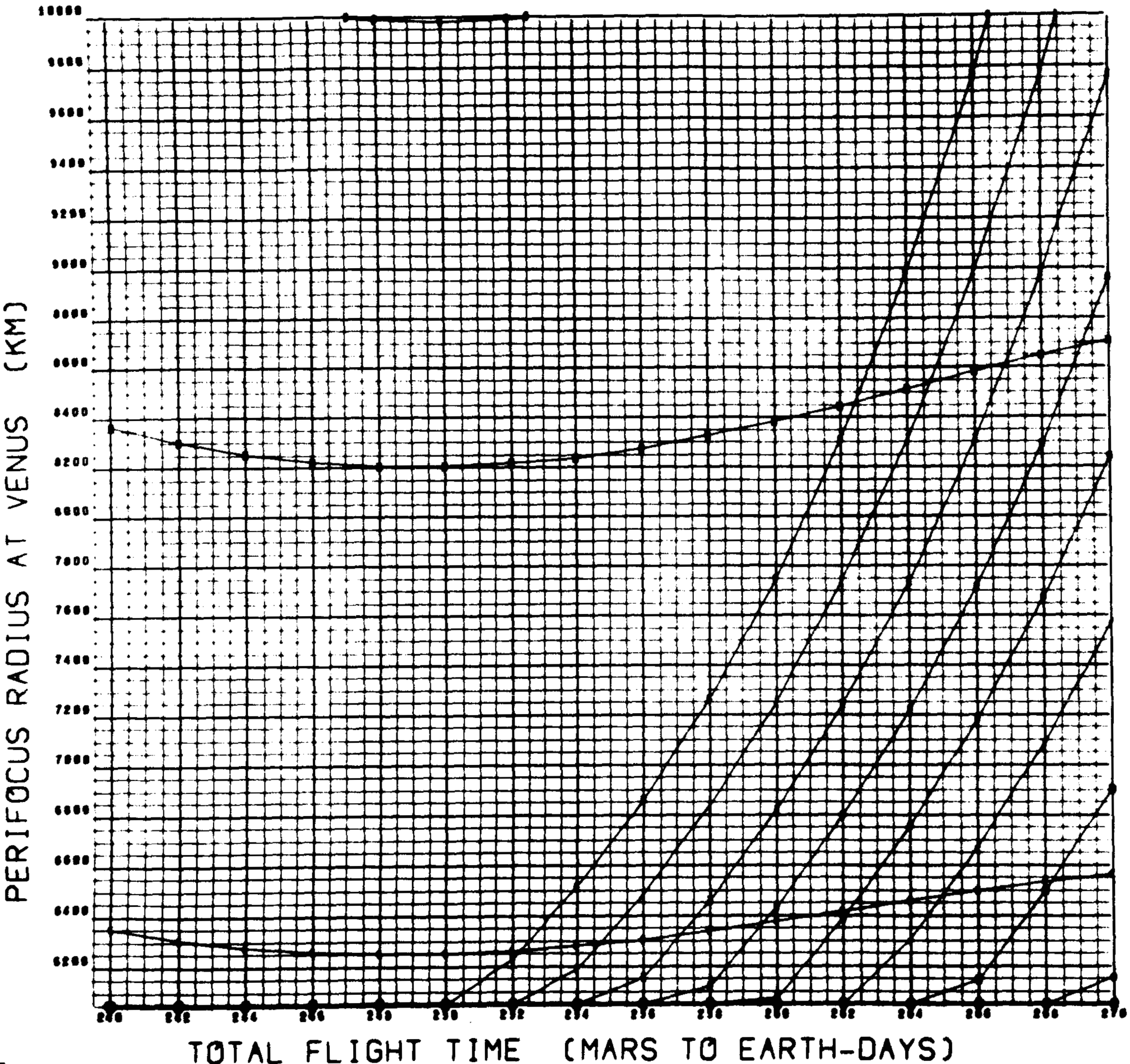

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
C3 - VENUS DEPARTURE (KM/SEC)**2
130
120
110
100
90
80
70
60
50
40
30
240
242
244
246
248
250
252
254
256
258
260
262
264
266
268
270
TOTAL FLIGHT TIME (MARS TO EARTH-DAYS)


CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978
DELTA V LAUNCH (100 KM ORBIT) (KM/SEC)
TOTAL FLIGHT TIME (MARS TO EARTH-DAYS)

CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978



CASE 13 RETURN FROM MARS VIA VENUS 1978
MARS LAUNCH DATE---- 02 MARCH, 1978